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1. Why there exists no constructive proof of Wiener's $1/f$ theorem?

The classical Wiener's $1/f$ theorem says that if a 2π periodic function f is the sum of an absolutely converging Fourier series and does not vanish then the Fourier series of $1/f$ is also absolutely converging. Very soon, this theoretical fact became an important tool of applied harmonic analysis, signal processing, and numerical analysis, but its very first proofs (N.Wiener and P.Levy, I.Gelfand) have contained no algorithm rules. Afterwards, several more "constructive" proofs have appeared (A.Calderon, P.Cohen, E.Bishop, D.Newman,...) but elusory elements persist in all proofs.

In this talk, after presenting one of the proofs mentioned, we discuss some recent results, which shed new light to the problem. Namely, we consider the problem of efficient control of the Wiener norm $\|f^{-1}\|$ in terms of the lower bound of the spectrum $inf_{x\in\mathbb{R}}|f(x)|$. It will be explained that such a control is possible if and only if the "condition number" $\delta = inf|f(x)|/\|f\|$ is bigger than a certain "critical constant" δ_1, in particular, if $\delta > 1/\sqrt{2}$.

No prerequisite knowledge of the subject is assumed.

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2. The Problem of Efficient Inversions in Fourier algebras

The problem is how to control the norms of inverses in terms of the "visible" lower spectral bounds of elements of function spaces or algebras. The generic example is the well-known Wiener's $1/f$ theorem for the algebra W of absolutely convergent Fourier series on the torus $\mathbb{T}$. Here, the entire spectrum $f(\mathbb{T})$ is well known (and this is trivial), but, what is less trivial, an efficient control of $\|f^{-1}\|$ is possible only if the "condition number" $\delta = inf|f(\mathbb{T})|/\|f\|$ is bigger than a certain "critical constant", say $\delta > 1/\sqrt{2}$. Many other spaces and algebras provide examples of very various behaviour with respect to such a problem of efficient control of inverses, from those allowing an estimate $\|f^{-1}\| \leq c(\delta)$ for every $\delta > 0$ (as for algebras of functions f having a fractional derivative $f^{(\alpha)}$ in W), and up to opposite examples, where no estimates are possible, even for $\delta = 1$ (as for the quotient algebra $H^\infty/\Theta H^\infty$ for a singular inner function Θ).

In this talk, I expect to consider the following topics.

1. Estimates of inverses in Fourier algebras on groups, unweighted (the group and semigroup algebras $\mathcal{F}L^1(G, dm)$, convolution measure algebras, etc) and weighted $\mathcal{F}L^p(G, wdm)$, as well as in some algebras of formal power series.
2. Norm controlled functional calculi, or quantitative problems for functions operating on a space or an algebra.
3. Inverses of Toeplitz and Wiener-Hopf operators.

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3. Asymptotics of condition numbers of large matrices and analytic capacities

The problem of bounding the condition numbers $CN(T) = \|T\| \cdot \|T^{-1}\|$ in terms of the spectral condition numbers $SCN(T) = \|T\| \cdot r(T^{-1})$, where $r(\cdot)$ stands for the spectral radius, is considered. We deal with classes of $N \times N$ matrices normalized by a specific functional calculus over a space (an algebra) of holomorphic functions A. It is shown that the norm of an inverse $\|T^{-1}\|$ of a matrix T of the class is determined by the analytic A-capacity of the spectrum of T. Mutually uniform asymptotics of the A-capacities of a "worst" n-points set are found as $n \longrightarrow \infty$ and/or $r(T^{-1}) \longrightarrow \infty$ for the scale of Besov spaces $B^s_{p,q}$ ($\sim n^s r(T^{-1})^{-n}$, where $n = deg(T)$) and for Beurling-Sobolev spaces $\mathcal{F}l^p(\mathbb{Z}_+, w_n)$.

If the times permits, a similar problem for Toeplitz, Wiener-Hopf and multiplication operators on the $\mathcal{F}l^p$ spaces will be discussed.

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Books: *"Selected Problems of Polynomial Approximation and Spectral Analysis"*, Proc. Steklov Math. Inst., vol.120 (1974) (Russian); English transl.: AMS, 1976.

"Lekzii ob operatore sdviga", "Nauka", Moscow, 1980 (Russian);

"Treatise on the Shift Operator. Spectral Function Theory", Springer-Verlag, Berlin etc., 1986.

"Operators, Functions, and Systems: an easy reading", Vol.1&2, Math. Monographs and Surveys, 92 and 93, Amer. Math. Soc., 2002.