Graduate Talk

What is the Atiyah-Singer index theorem about?

I shall try to explain the connections between linear elliptic partial differential equations and topology that led to the formulation and proof of the famous Atiyah-Singer index theorem. I shall include all the necessary definitions during the talk (or at least informal versions of them) and as a result the lecture will be accessible to all.

Colloquium I

Weyl's theorem, noncommutative geometry and index theory

Over the past twenty years, Alain Connes has transformed the idea that noncommutative algebras should be regarded as function algebras on "noncommutative spaces" into a substantial mathematical theory. His noncommutative geometry is particularly concerned with aspects of the algebra-space correspondence that on the algebra side involve Hilbert space methods, especially the spectral theory of operators on Hilbert space. Motivation for this comes from a long series of discoveries, perhaps beginning with Hermann Weyls asymptotic formula connecting geometry to the spectral theory of the Laplace operator. I shall try to present some of this background along with some of the basic constructions of noncommutative geometry, including concepts related to the Atiyah-Singer index theorem.

Colloquium II

Dimensions and C^*-algebras

I shall describe some of the ways in which algebraic topology enters into the study of algebras of operators on Hilbert space around the problem of associating numerical "dimensions" to the ranges of projection operators. My account will begin with the pioneering work of Murray and von Neumann (where topology does not play an immediate role) and end with the Baum-Connes conjecture and some of its connections to geometry and representation theory. These ideas represent some of the topological aspects of Alain Connes' noncommutative geometry program.