

#1 (Tuesday)

### Games in Banach spaces

**Abstract:** Let  $X$  be a separable infinite dimensional Banach space. We explore the theory and consequences of certain games on Banach spaces of the following type. Player  $S$ , the subspace player, chooses  $X_1 \subseteq X$ , an infinite dimensional subspace (possibly restricted to be of finite co-dimension) and player  $V$ , the vector player, chooses  $x_1 \in S_{X_1}$  = the unit sphere of  $X_1$ . Plays alternate in this fashion. Plays may stop at a prescribed integer  $n$  or may continue forever. What sort of sequences  $(x_i)$  can  $V$  be assured of choosing? What class of  $(x_i)$ 's can  $S$  force  $V$  to choose? If  $S$  can force a certain outcome, e.g.,  $\|\sum a_i x_i\| \sim (\sum |a_i|^p)^{1/p}$ , what does this imply about the structure of  $X$ ?

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#2 (Thursday)

### Coefficient Quantization in Banach spaces

**Abstract:** The Banach space  $c_0$  has the following property. If  $(e_i)$  is the unit vector basis for  $c_0$  then for all  $\varepsilon > 0$  there exists  $\delta > 0$  so that for all  $x = \sum_F a_i e_i \in c_0$ ,  $|F| < \infty$ , then there exists  $y = \sum_F n_i \delta e_i$ ,  $n_i \in \mathbb{N}$ , with  $\|x - y\| \leq \varepsilon$ . (In fact  $\delta = 2\varepsilon$  works.) We consider the problem of what other bases  $(e_i)$  have this property and what spaces have a basis with the property. We also consider the same question for systems weaker than bases and for variations of the property. For example every  $X$  contains a discrete subgroup ( $\varepsilon$ -separated) which is  $M$ -dense in  $X$ .

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#3: (Friday)

### Small subspaces of $L_p$

**Abstract:** We discuss the "small" subspaces of  $L_p[0, 1]$ , namely  $\ell_p$ ,  $\ell_2$ ,  $\ell_p \oplus \ell_2$  and  $(\sum \ell_2)_{\ell_p}$ , with regard to the problem: Given a subspace  $X$  of  $L_p$  when does  $X$  contain or embed into one of the above spaces. We'll review some of the 40+ year history and also give some recent new results.