The common theme of these lectures is interaction between cluster algebras and quiver representations. The theory of cluster algebras was initiated by S.Fomin and myself in 2002, and has been rapidly developed by many people. A lot of information about this theory and its connections with various branches of mathematics is available online at the Cluster Algebras Portal (http://www.math.lsa.umich.edu/∼fomin/cluster.html) created and maintained by S.Fomin. We attempt to make the lectures reasonably independent and self-contained. All necessary background on cluster algebras and quiver representations will be presented from scratch.

Lecture 1: Graduate

Birational recurrences, Laurent polynomials and representations of the Kronecker quiver

An important feature of cluster algebras is appearance of a large class of birational recurrences whose solutions rather unexpectedly turn out to be Laurent polynomials in the initial data. In this lecture we discuss the first non-trivial example of this "Laurent phenomenon." Namely, we consider the sequence (x_m) satisfying the recurrence $x_{m-1}x_{m+1} = x_m^2 + 1$; the Laurent phenomenon then asserts that each x_m is a Laurent polynomial in x_1 and x_2 with integer coefficients. As discovered by P.Caldero and F.Chapoton, the coefficients of these polynomials have a nice geometric interpretation as Euler-Poincare characteristics of certain quiver Grassmannians. Without developing general machinery, we explain their result for the above sequence, and use it to find a simple explicit combinatorial expression for the corresponding Laurent polynomials (this is recent joint work with P.Caldero).

Lecture 2: Colloquium

Introduction to cluster algebras

In this lecture we introduce cluster algebras and discuss their main structural properties (including the Laurent phenomenon) and the classification of cluster algebras of finite type due to S.Fomin and myself. If time allows, we will discuss the above mentioned result by Caldero-Chapoton and its generalization due to P.Caldero and B.Keller.

Lecture 3: Colloquium

Mutations for quivers with potentials and their representations

This lecture is based on a joint work in progress with H.Derksen and J.Weyman. We obtain a far-reaching generalization of Bernstein-Gelfand-Ponomarev reflection functors in the theory of quiver representations. Namely, the classical reflection functors are defined only at a source or a sink of the quiver in question. We introduce a general class of quivers with relations (given by non-commutative Jacobian ideals in the path algebra) and define the mutations at arbitrary vertices for these quivers and their representations. If the vertex in question is a source or a sink, our mutations specialize to reflection functors.