From bounded symmetric domains to their compact duals – rigidity by means of rational curves

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Abstract.   This is an exploratory lecture concerning two different fields of research: Several Complex Variables and Algebraic Geometry. More specifically, it concerns Function Theory on bounded symmetric domains on the one side, an important subject area in Several Complex Variables pioneered by the late Professor Lo-Keng Hua, and Fano manifolds on the other side, a subject area in Algebraic Geometry of great current interest.

The ground-breaking work of Shigefumi Mori yielded rational curves on a projective manifold whose tangent bundle is not numerically effective, leading by Miyaoka-Mori to the proof that Fano manifolds are covered by rational curves. In the last ten years or so, the speaker and Jun-Muk Hwang have collaborated to develop a geometric theory on Fano manifolds basing on the study of vectors tangent to free rational curves of minimal degree, called *varieties of minimal rational tangents* (VMRTs). One of the key principles established in the geometric theory is the Cartan-Fubini Extension Principle, according to which a local biholomorphic map between two Fano manifolds of Picard number 1 extends automatically to a global biholomorphism unless the VMRT at a general point is a finite union of projective linear subspaces.

Recently, in examining the problem of rigidity of proper holomorphic maps between irreducible bounded symmetric domains of rank ≥ 2, a problem posed by the speaker and resolved by Tsai using methods of Complex Differential Geometry and Lie Theory, the speaker has found an intimate link between the study of proper holomorphic maps and the geometric theory of Fano manifolds based on VMRTs. This link is realized by identifying a bounded symmetric domain as a domain in its compact dual, e.g., a Type-I domain as a domain on the Grassmann manifold. Harmonic Analysis on bounded symmetric domains leads to differential constraints which translate the rigidity problem to a question of characterizing *non-equidimensional* local holomorphic maps which respect VMRTs. The speaker has settled the latter problem in terms of a *relative* differential projective-geometric criterion on VMRTs, and the approach opens up a direction of research starting with identifying bounded domains with interesting underlying geometric structures inherited from VMRTs of dual quasi-projective manifolds. This should at least include certain classes of bounded homogeneous domains from the work of Pyatetskii-Shapiro but should also include interesting examples which are not homogeneous. In this talk we examine by way of a concrete example how the link between the two fields of research as mentioned is established.