THE DIRICHLET PROBLEM IN CALIBRATED AND RIEMANNIAN GEOMETRY

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Lecture 1.
Calibrations and Geometric Plurisubharmonic Functions

This lecture will be an elementary introduction to calibrated geometries and the Dirichlet probelm for "harmonic" functions in these geometries. It will begin with a general discussion of the classical Dirichlet problem in $\mathbf{R}^n$ and present a unified approach to solving this problem for basic equations such as Laplace's equation and the real and complex Monge-Ampère equations. Then the elementary theory of calibrations will be presented with a number of examples including: Kähler, quaternionic Kähler, Special Lagrangian (on Calabi-Yau spaces), and associative-coassociative (on G_2-spaces). In each calibrated geometry there is a notion of harmonic functions with a naturally associated Dirichlet problem. The solution of this general problem will be discussed and related to the classical examples above. The end of this lecture will summarize a broad generalization of these ideas to more general differential equations in riemannian geometry.

Lecture 2.
Dirichlet Duality in $\mathbf{R}^n$

This lecture will discuss the Dirichlet problem for fully nonlinear partial differential equations on domains in $\mathbf{R}^n$. It will include a notion of duality for such equations. We will treat highly degenerate phenomena such as the intermediate branches of the Monge-Ampère and Special Lagrangian potential equation.

Lecture 3.
Dirichlet Duality on Riemannian Manifolds.

I will discuss the Dirichlet problem for G-universal equations on riemannian manifolds with a topological G-structure. This includes, for example, (all branches of) the complex Monge-Ampère equation on an almost complex hermitian manifold. Comparison results for such equations will be given, and boundary convexity conditions needed for existence will be presented.

This is a report on recent joint work with REESE HARVEY.