Lecture 1 (graduate)

Quantum Unique Ergodicity and Number Theory

A fundamental problem in the area of quantum chaos is to understand the distribution of high eigenvalue eigenfunctions of the Laplacian on certain Riemannian manifolds. A particular case which is of interest to number theorists concerns hyperbolic manifolds arising as a quotient of the upper half-plane by a discrete "arithmetic" subgroup of $SL_2(\mathbf{R})$ (for example, $SL_2(\mathbf{Z})$, and in this case the corresponding eigenfunctions are called Maass cusp forms). In this case, Rudnick and Sarnak have conjectured that the high energy eigenfunctions become equi-distributed. I will discuss some recent progress which has led to a resolution of this conjecture, and also on a holomorphic analog for classical modular forms. I will not assume any familiarity with these topics, and the talk should be accessible to graduate students.

Lecture 2 (colloquium)

Weak subconvexity for central values of L-functions

L-functions are analytic objects encoding data of interest to number theorists; for example, the distribution of prime numbers is encoded by the Riemann zeta-function. One important problem is to obtain upper bounds for L-functions at special points. While the Generalized Riemann Hypothesis would give very good bounds for these values, for various applications even a small improvement over the "trivial" bound is enough. This is called the "subconvexity problem", and is still largely open. I'll explain how such bounds shed light on arithmetical problems, and report on progress towards a weak version of subconvexity which already is enough for some applications.

Lecture 3 (colloquium)

The distribution of values of zeta and L-functions

I will survey what is known and expected about the distribution of values of L-functions. In particular, I will try to explain probabilistic models that describe the behavior of these functions. At the center of the critical strip, the probabilistic models arise from random matrix theory, and I will discuss the Keating-Snaith conjectures for moments of L-functions, and recent progress.