This series of talks gives fundamental ideas, methods and results in analysis on covering spaces. The main focus is put on spectral analysis on covering graphs over a finite graph. Analogy to algebraic number theory and algebraic geometry turns out to be useful. Some results for manifolds will be also discussed.

Lecture 1 - graduate talk

Analysis on covering spaces

Starting with a quick review of covering spaces, we will handle Laplacians on covering manifolds and graphs. The notion of twisted Laplacians plays a significant role in our discussion. Indeed, this provides us with a unified tool to construct isospectral manifolds and graphs, to establish a relation between fundamental groups and spectra, and to give a family of expanders. Cayley graphs and crystal lattices are treated as examples of covering graphs.

Lecture 2 - colloquium

Abel-Jacobi maps and Albanese maps in graph theory

It is observed that a graph version of Abel-Jacobi maps is related to Albanese maps of graphs, which is a canonical harmonic map into a torus with a flat metric. The notion of Albanese maps (and its generalization) is used for geometric crytallography. For an illustration, we will treat the *diamond twin* which has the same property of symmetry as the diamond crystal and has been pinned down in the study of random walks on crystal lattices.

Lecture 3 - colloquium

Large deviation asymptotics of the heat kernels on periodic manifolds

As one of long-time behaviors of the heat kernel, we look at a large deviation asymptotic of the one on a periodic manifold, an infinite fold abelian covering manifold over a closed manifold. Twisted Laplacians associated with non-unitary characters play an essential role in our discussion. A discrete version (random walks on crystal lattices) is also mentioned.