THE UNCERTAINTY PRINCIPLE AND UNIQUENESS SETS FOR THE KLEIN-GORDON EQUATION

ABSTRACT. In this talk, we will see how one-to-one compressions of composition operators on $L^1[-1,1]$ applies to show that the system

$$e^{\pi i\alpha nt}, \quad e^{i\pi\beta nt} \quad n = 0, 1, 2, \dots ,$$

where α and β are positive numbers, is weakly dense on $L^\infty(\mathbb{R})$ if and only if $\alpha\beta \leq 1$. This problem can be stated in terms of the solution of a version of the 1D Klein-Gordon equation. In fact, if a bounded Borel measure μ supported in a curve $\Gamma \subset \mathbb{C}$, which is absolutely continuous with respect to the arc length, and whose Fourier transform $\widehat{\mu}$ vanishes on a set $\Lambda \subset \mathbb{C}$, must be athomatically the zero measure, (Γ, Λ) is called a Heisemberg uniqueness pair. When Γ is the hyperbola $x_1x_2 = 1$, and Λ is the lattice-cross

$$\Lambda = (\alpha\mathbb{Z} \times \{0\} \cup (\{0\} \times \beta\mathbb{Z}),$$

then (Γ, Λ) is Heisemberg uniqueness pair if and only if $\alpha\beta < 1$; in this situation $\widehat{\mu}$ solves the version of the Klein-Gordon equation.

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