

The Gaussian Correlation Conjecture

The standard Gaussian measure on \mathbb{R}^n is given by

$$\gamma(A) = \gamma_n(A) = \frac{1}{(2\pi)^{n/2}} \int_A e^{-|x|^2/2} dx.$$

A general mean zero Gaussian measure on \mathbb{R}^n is just a linear image of the standard Gaussian measure.

Conjecture (GCC). . *For any $n \geq 1$, if μ is a mean zero, Gaussian measure on \mathbb{R}^n , then for any $A, B \subseteq \mathbb{R}^n$, which are convex and symmetric about the origin,*

$$\mu(A \cap B) \geq \mu(A)\mu(B).$$

A much weaker version of the problem was considered in 1955 and appeared in its present form about 1970. The original application was to (multidimensional) confidence regions in Statistical problems.