Links between low order DG methods and finite difference schemes for the Stokes problem

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Kanschat (2007) has shown that with a proper quadrature, the LDG method can be made algebraically equivalent to the MAC scheme. The LDG method is based on the H^{div}-conforming Raviart-Thomas RT_0 element. In the present study we show that the Nédélec space of first kind, within the LDG setting, can yield a scheme which is also algebraically equivalent to the MAC method. In addition, we show that a piecewise constant approximation for the velocity can yield two other popular finite difference schemes proposed several decades ago.

These observations allow to use the LDG setting to generalize the corresponding finite difference methods to unstructured grids or grids with non-matching nodes. The generalization of the first scheme to 3D is straightforward. The generalization of the second scheme to 3D involves degrees of freedom for the velocity on the edges rather than faces of the elements and follows the same idea as the generalization of the Nédélec elements.

In addition, we also propose a procedure for constructing of a divergence free basis in the case of Nédélec approximation. The numerical tests show that, as it can be expected, the resulting linear system from the Stokes problem has a biharmonic conditioning. However, in case of large Reynolds numbers, this system may be preferable to solve and therefore, it can be a viable alternative to Uzawa iterations or projection. This basis can also be used for the resolution of the Maxwell equations in the limit of vanishing wave numbers. The same procedure can be employed for the construction of a divergence free basis in the case of Raviart-Thomas elements.