NUMERICAL ANALYSIS SEMINAR
4:00-5:00pm, October 9, 1996
503 Blocker

Closed Linear One-Cell Functional Methods for the Two Dimensional Transport Equation

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Abstract. We present a general formulation of methods for the solution of the two dimensional monoenergetic transport problem, which for a specific direction has the form $\mu\frac{\partial\psi}{\partial x}+\eta\frac{\partial\psi}{\partial y}+\sigma\psi=Q$. Here ψ is the angular flux in the given direction with direction cosines μ and η, σ is the total cross section, and $Q=q+S$, where q is an expression which accounts for (linear) scattering and S denotes the sources. Specifically, we extend Nelson's earlier work on closed linear one-cell functional methods (CLOF) for the one dimensional monoenergetic problem. The spatial domain for the problem is partitioned by a rectangular mesh of cells over which σ and the scattering kernel in q are constant. With respect to each cell C, two sets of linear functional, one set for cell surfaces and the other for cell volumes, are introduced in such a way that the method may be described to explicitly relate approximations of the cell exiting fluxes to the functions of the entering fluxes and the volume functions over the cell. As such these are a linear one-cell functional methods (LOF). The closure approximations are constructed from an additional set of linear functionals derived in principle from the original equation and in the interest of computational economy involve the same variables used in the LOF method. We begin with the analysis of the classical nodal methods. These methods are typified by tranverse integrations across the two coordinate directions to reduce the PDE to a set of ordinary differential equations. There must be at least two such equations for each direction, but there can be many if multiple tranverse integrations against some set of basis functions is performed. What we demonstrate is that most of the methods of solution that begin from this system, whether S_N methods or others, can be cast into the general framework. Moreover, such methods can be attributed as consistent, convergent within this framework, Order can also be ascribed.