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Analysis and discretization of a time-dependent grade-two fluid model in a plane domain.

This is common work with M. Saadouni.

We consider the following time-dependent grade-two fluid model in a plane Lipschitz domain:

$$\frac{\partial}{\partial t}(\boldsymbol{u} - \alpha\Delta\,\boldsymbol{u}) - \nu\Delta\,\boldsymbol{u} + \mathbf{curl}(\boldsymbol{u} - \alpha\Delta\,\boldsymbol{u}) \times \boldsymbol{u} + \nabla\, p = \boldsymbol{f}\,,$$

together with the incompressibility condition:

$$\mathrm{div}\,\boldsymbol{u} = 0\,,$$

and adequate boundary and initial conditions. We prove existence of a weak solution without restriction on the domain and uniqueness of the solution in a convex polygon. The method of proof is constructive. It is based on splitting the problem into a generalized time-dependent Stokes system of equations and a transport equation obtained by taking the curl of the equation of motion. Both are discretized in time, but not in space. Owing to the particular structure of the transport equation in two dimensions and to sharp results on the solution of this equation, there are enough a priori estimates to pass to the limit and establish existence of a solution. This extends previous results of Cioranescu and Ouazar on a simply-connected domain with a smooth boundary. Uniqueness requires more regularity on the solution and is established on a convex polygon or a smooth domain.

The semi-discrete scheme is then discretized in space, thus giving rise to a linearized fully discrete finite element scheme. A priori error estimates are derived under mild restrictions on the ratio of the time step and mesh size in space.