NUMERICAL ANALYSIS SEMINAR

Finite-element discretization of a two-dimensional, grade-two fluid

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3:00-4:00pm
503 Blocker

Abstract

We discretize a simplified model of grade-two fluid in two-dimensions: Find a velocity vector $\mathbf{u}$ and a scalar pressure p such that

$$-\nu\,\Delta\,\mathbf{u} + \mathbf{curl}(\mathbf{u} - \alpha\,\Delta\,\mathbf{u}) \times \mathbf{u} + \nabla\,p = \mathbf{f} \quad \text{in } \Omega\,,$$

$$\operatorname{div}\mathbf{u} = 0 \text{ in } \Omega\,,$$

$$\mathbf{u} = \mathbf{0} \text{ on } \partial\Omega\,.$$

Here Ω is a bounded polygonal domain, $\mathbf{f}$ a given exterior force, $\nu > 0$ a given viscosity coefficient and $\alpha \neq 0$ a given normal stress modulus coefficient. This problem models a water solution of polymers, which is a non-Newtonian fluid. It has at least one solution, if $\mathbf{f}$ belongs to $H(\operatorname{curl};\Omega)$. Introducing the auxiliary variable $z = \operatorname{curl}(\mathbf{u} - \alpha\,\Delta\,\mathbf{u})$, we write the problem equivalently in the form

$$-\nu\,\Delta\,\mathbf{u} + \mathbf{z} \times \mathbf{u} + \nabla\,p = \mathbf{f} \quad \text{in } \Omega\,,$$

$$\nu z + \alpha\mathbf{u} \cdot \nabla\,z = \operatorname{curl}\,(\nu\mathbf{u} + \alpha\mathbf{f}) \text{ in } \Omega\,,$$

$$\operatorname{div}\mathbf{u} = 0 \text{ in } \Omega\,,$$

$$\mathbf{u} = \mathbf{0} \text{ on } \partial\Omega\,.$$

We discretize $\mathbf{u}$ and p in a pair of stable finite-element subspaces of $H_0^1(\Omega)^2 \times L^2(\Omega)$, such that $\mathbf{u}_h$ is exactly divergence-free, an d z_h in a finite-element subspace of $H^1(\Omega)$. We show that, without restricti on on the data, the discrete problem has at least one solution that converges to a solution of the original problem. Error estimates are established under suitable assumptions on the data.