Parametric argument principle and detecting CR functions and manifolds

Mark Agranovsky

Bar-Ilan University and Texas A&M University

Abstract

The classical argument principle implies that no homologically trivial smooth loop in $\mathbb{C}^n$ can bound a nontrivial (i.e., one-dimensional) complex variety.

On analytic language, this means that if f is a mapping of the unit disc Δ, holomorphic in its interior and smooth on its closure, and the boundary image $f(\partial\Delta)$ is a homologically trivial closed curve, then f degenerates dimensionally (is constant). We study an analogous phenomenon for families of holomorphic mappings $f_t : \Delta \mapsto \mathbb{C}^n$, parametrized by points t from a real analytic manifold, and give conditions when the homological degeneracy of the boundary image $\cup_t f_t(\partial\Delta)$ implies the dimensional collapse of the interior image $\cup_t f_t(\Delta)$.

The obtained result implies lower estimates, in terms of moment conditions on families of curves or, alternatively, in terms of families of attached analytic discs, of CR dimensions of real submanifolds in $\mathbb{C}^n$. In particular, in the case when the submanifold is a graph, these estimates contain solutions, in generic case, of two problems on CR functions (strip-problem and Globevnik-Stout conjecture) which were solved earlier by different authors only for special cases. Also a generalization of a result of Dolbeault-Henkin-Dinh about borders of analytic varieties follows.