March, 1991

Syllabus
Qualifying Examination
Complex Analysis

1. ARITHMETIC, GEOMETRY, AND TOPOLOGY OF THE COMPLEX NUMBERS: Field operations; stereographic projection; spherical metric; simple and multiple connectivity.
2. ANALYTIC FUNCTIONS: Cauchy-Riemann equations; power series, harmonic functions.
3. COMPLEX INTEGRATION: Cauchy's theorem; Goursat's proof; Cauchy's integral formula; residue theorem; computation of definite integrals by residues.
4. CONFORMAL MAPPING: linear fractional transformations and cross ratio; mappings by elementary functions; Riemann mapping theorem.
5. SINGULARITIES: classification of isolated singularities; Laurent series; Casorati-Weierstrass theorem; Picard's theorems.
6. GEOMETRIC FUNCTION THEORY: winding numbers and the argument principle; open mapping theorem; maximum principle; Schwarz lemma; three-circles theorem.
7. ANALYTIC CONTINUATION: Schwarz reflection principle; continuation along a path; monodromy theorem.
8. CONVERGENCE AND APPROXIMATION: normal families; Hurwitz's Theorem; Runge's Theorem; Mittag-Leffler's Theorem; infinite products; factorization theorems of Weierstrass and Hadamard.

References:

- *Functions of One Complex Variable*, second edition, John B. Conway, Springer-Verlag, 1978.
- *Invitation to Complex Analysis*, Ralph P. Boas, Random House, 1987.
- *Complex Analysis*, third edition, Lars V. Ahlfors, McGraw-Hill, 1979.
- *An Introduction to Classical Complex Analysis*, Volume 1, Robert B. Burckel, Academic Press, 1979.