Equilibrium Equations: Polynomials and Polytopes

Sagar Bhatt, Jeff Pattillo, Katie Uttke

"The concept of a Nash equilibrium n-tuple is perhaps the most important idea in non-cooperative game theory. ... Whether we are analyzing candidates' election strategies, the causes of war, agenda manipulation in legislatures, or the actions of interest groups, predictions about events reduce to a search for and description of equilibria. Put simply, equilibrium strategies are the things that we predict about people."
- P. Ordeshook

In 1950, John Nash's paper, entitled, "Equilibrium Points in N-Person Games," fundamentally changed economic theory. Using mathematical theory, specifically polynomial system solving, along with economic theory, Nash discovered that the solutions to a set of polynomial systems yielded a new type of equilibrium, creatively named, Nash Equilibrium. Given a non-cooperative n player game, in which each player must choose a strategy, the outcome will be determined by all players' strategies and their given corresponding payoff matrices. Each player strives to obtain the outcome which is most profitable to them. A Nash Equilibrium is a set of optimal strategies, one for each player, in which no player can, by changing their strategy, increase their payoff, given the other players' strategies remain fixed. i.e. a strategy combination with the property that no player can gain by individually deviating from it. Thus, Nash Equilibria are the most desirable outcomes for all players in any game. We will illustrate what Nash Equilibria are, how they can be solved, and a bound on the number that exist. To do this, we will apply techniques from algebraic geometry. This will show the intimate connection between polyhedra and Nash equilibria.

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