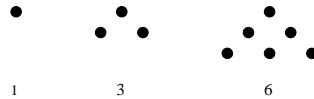


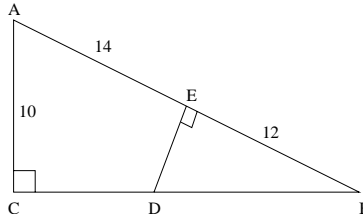
BEST STUDENT EXAM
TAMU High School Mathematics Contest
November 2001

1. Find the area of an equilateral triangle which is inscribed in a circle of area π .
2. The probability that Patty passes a driving test is p and the probability that she fails is $6p^2$. Find the value p .
3. A cylindrical can without a top holds 100 cm^3 of a liquid when completely full. The radius of the base of the can is r cm. Express the surface area of the can in square centimeters as a function of r .
4. If $x, 2x+2, 3x+3, \dots$ are nonzero terms in a geometric progression (geometric sequence), what is the fourth term?
5. The numbers in the figure shown below are called *triangular numbers*:



- Let a_n be the n th triangular number, with $a_1 = 1, a_2 = 3, a_3 = 6$ etc. What is $a_n^2 - a_{n-1}^2$?
6. Let P be the point $(3, 2)$. Let Q be the reflection of P about the x -axis, R the reflection of Q about the line $y = -x$ and S the reflection of R through the origin. $PQRS$ is a convex quadrilateral. Find its area.
 7. Find the minimum value of $f(x) = 2|2x - 1| - |3x - 1| + |4x - 3|$ on the interval $[0, 1]$.
 8. A regular dodecagon (12 sides) is inscribed in a circle of radius r inches. What is the area of the dodecagon in square inches?
 9. Let $S = \{a, b, c, d, e\}$. Find $\sum_{A \subseteq S} n(A)$, where $n(A)$ is the number of elements in the set A .
 10. The sum of two numbers is 4 and their product is 1. Find the sum of their cubes.

11. Triangle ABC is a right triangle, D is a point on the leg BC and E is the foot of the perpendicular from D to the hypotenuse AB . If the segments AC , AE and EB are 10, 14 and 12 respectively, find the length of segment BD .



12. The batting average for a baseball player is determined by dividing his total number of hits for the season by his total number of official at bats for the season. A baseball player had an average of 0.250 prior to his game yesterday. The player had 0 hits in 4 official at bats in his game yesterday and his average dropped to 0.2475. How many hits does this player have for the season?

13. If f is twice continuously differentiable, find $\lim_{h \rightarrow 0^+} \frac{f(a + \sqrt{h}) - 2f(a) + f(a - \sqrt{h})}{h}$.

14. Let w be a solution to $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$. Find w^{14} .

15. Let s be the limiting sum of the series

$$4 - \frac{8}{3} + \frac{16}{9} - \frac{32}{27} + \dots$$

Then s equals ?

16. Any five points are taken inside or on a square of side length 1. Let a be the smallest possible number with the property that it is always possible to select one pair of points from these five such that the distance between them is less than or equal to a . What is the value of a ?

17. Let n be the number of pairs of numbers b and c such that $3x + by + c = 0$ and $cx - 2y + 12 = 0$ have the same graph. Find n .

18. Find three integers which form a geometric progression if their sum is 21 and the sum of their reciprocals is $\frac{7}{12}$.

19. $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 3x} - \sqrt{x^2 - 3x}}{\sqrt{x^2 + 9x} - \sqrt{x^2 - 9x}} =$

20. Suppose that x satisfies the equation $\sin 2x - \cos 3x = 0$. Find the smallest possible value of $\cos x$.

21. Among all ordered pairs of real numbers (x, y) which satisfy

$$x^4 + y^4 = x^2 + y^2,$$

what is the largest value of x ?