

EF Exam

1. Find all real values of x which satisfy the equation

$$\sqrt{x} - \frac{6}{\sqrt{x}} = 1$$

2. Find the point on the line $3x + 4y = 10$ which is closest to the origin.

3. Find all values of a such that $\lim_{x \rightarrow -2} \frac{x^2 + ax - 6}{x^2 + x - 2}$ exists and is finite.

4. Let S be the set of all x such that $x^4 - 10x^2 + 9 \leq 0$. Find the maximum value of $f(x) = x^3 - 12x$ on S .

5. Water flows into a hemispherical bowl 16 inches in diameter at a rate of 2 cubic inches per second. How fast is the water level rising when the water level is halfway to the top?

6. Compute $\log_6 14 - 2 \log_6 3 - \log_6 28 - \log_6 12$

7. Find all functions f such that

$$(f(x))^2 = 2003 + \int_0^x [(f(t))^2 + (f'(t))^2] dt$$

8. The *normal line* to a curve C at a point P is the line that passes through P and is perpendicular to the tangent line of C at P . If the normal lines of the curve $y = x^2$ at the points where $x = a$, $x = b$, and $x = c$ all intersect at the same point, compute $a + b + c$.

9. Both x and y are positive numbers less than 2. Every positive number less than 2 is equally likely to be the value of x , and every positive number less than 2 is equally likely to be the value of y . What is the probability that x and y differ by less than 1?

10. Compute

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x \sin(2x)}$$

11. The decimal $0.\overline{3} = 0.333333 \dots$ in base 10 is equal to the fraction $\frac{1}{3}$. What fraction is $0.\overline{3} = 0.333333 \dots$ in octal (base 8) equal to?

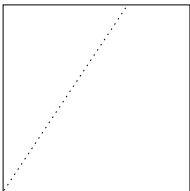
12. Let f and g be differentiable functions with some values of the functions and their derivatives given in the chart below. What is the derivative of $f(g(x))$ at $x = 2$?

x	1	2	3	4
$f(x)$	2	4	3	1
$f'(x)$	4	3	2	1
$g(x)$	3	1	4	2
$g'(x)$	2	4	1	3

13. Compute

$$\lim_{n \rightarrow \infty} \sum_{i=1}^{\infty} \frac{1}{\sqrt{n}\sqrt{n+i}}$$

14. A student is standing at one end of a square swimming pool which is 150 feet long. The student can run at a speed of 20 feet per second and swim at a speed of 4 feet per second. The student wants to get to the opposite corner of the pool as fast as possible by swimming to a spot on the opposite end of the pool and running the rest of the way. How far should she run?



15. Find all values of c such that the equations

$$\begin{aligned} & \cdot \quad x - y = 2 \\ & \cdot \quad cx + y = 3 \end{aligned}$$

have positive solutions for both x and y .

16. Find the x -coordinate of all points on the graph of $f(x) = \frac{x}{x+1}$ where the tangent line also passes through the point $(1, 2)$.

17. Compute

$$\int_0^{2\sqrt{2}} x \, dx + \int_{2\sqrt{2}}^4 \sqrt{16 - x^2} \, dx$$