

## DE Exam

1. A rectangle has perimeter 22 and area 30. What is the length of the rectangle's diagonal?
2. Given that  $\log_b 2 + \log_b 4 = \frac{1}{3}$ , find  $b$ .
3. If  $f$  is a function such that for all positive integers  $n$ ,

$$f(n+1) = \frac{2f(n) + 1}{2}$$

and  $f(1) = 1$ , find  $f(2357)$ .

4. If  $0^\circ < A < 90^\circ$ , and  $\sin A = 0.1$ , find the value of

$$\log_{10}(\tan A) + \log_{10}(\cos A).$$

5. Suppose  $f$  is a function satisfying

$$2f(x) - f(1/x) = x^2, \text{ for } x \neq 0.$$

Find  $f(2)$ .

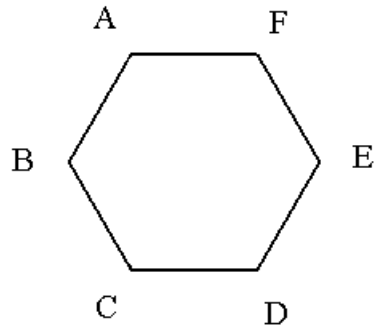
6. If  $f(x) = \begin{cases} 0 & \text{if } x \text{ is a rational number} \\ 1 & \text{if } x \text{ is an irrational number} \end{cases}$ ,  
find the value of  $2f(\sqrt{2}) - 3f[f(\sqrt{2})]$ .

7. Let  $S$  be a square centered at the point  $(-1, 3)$  in the  $xy$ -plane. If all of the vertices on the square lie either on or below the line  $3x + y = 17$ , find the maximum possible area of the square.
8. Find the sum of the series

$$\sin \frac{\pi}{2} + 2 \sin \frac{2\pi}{2} + 3 \sin \frac{3\pi}{2} + \cdots + 51 \sin \frac{51\pi}{2}.$$

9. Suppose a rectangular solid has a base of area 6 square inches, a side of area 8 square inches, and a front of area 12 square inches. What is the volume of the solid?

10. Each side of the regular hexagon ABCDEF is 10 cm long. What is the area of the hexagon?



11. Find the area of the triangle with vertices given by  $(-2, 1)$ ,  $(0, 3)$  and  $(3, -4)$ .
12. Find all  $x$  such that  $\frac{3}{2-x} < 1$ . Write your answer using interval notation.
13. Find the length of the common chord of the two circles whose equations are  $x^2 + y^2 = 4$  and  $x^2 + y^2 - 6x + 2 = 0$ .
14. For which values of  $a$  will the system

$$ax + 2y + 3z = 1$$

$$4x + 5y + 6z = 2$$

$$7x + 8y + 9z = 3$$

have a unique solution?