

CD EXAM

1. For what value(s) of the number a do the equations

$$\begin{aligned}x^2 - ax + 1 &= 0 \\x^2 - x + a &= 0\end{aligned}$$

have a common real solution?

2. Find the domain of

$$f(x) = \frac{2}{[x]^2 + [x] - 56}$$

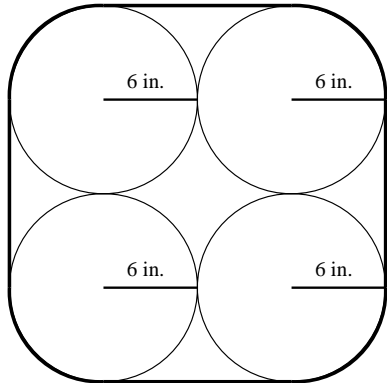
where $[]$ is the greatest integer function.

3. Let

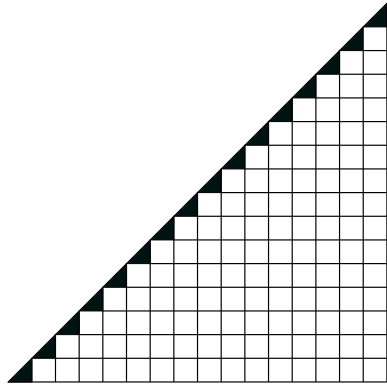
$$f(x) = \frac{cx}{2x + 3}$$

with $x \neq -3/2$. Find all values of c , if any, for which $f(f(x)) = x$ for all $x \neq -3/2$.

4. Find the y -coordinate of the point on the y -axis which is equidistant from $(5, -5)$ and $(1, 1)$.
5. A cube of volume 216 cubic inches is inscribed in a sphere. What is the surface area of the sphere?
6. An elastic band is placed around the top of 4 circular cans, each with a radius of 6 inches. Find the length of the stretched band.



7. Find two distinct (complex) numbers each of which is the square of the other.
8. A positive integer x is chosen with $10 \leq x \leq 99$. If x is divided by the sum of its digits, how small can the result be?
9. The square of the sum of two numbers is 64 and the sum of the squares of the two numbers is 34. Find the product of the two numbers.
10. Find the ratio of the area of the shaded area to the unshaded area.



11. Pick any 5 points in the plane (p , q , r , s , and t) and draw the line segments pq , qr , rs , st , and tp (they are allowed to cross). If a line l is drawn which goes through all 5 of these line segments, how many of the 5 original points must lie on l ? In other words, can the 5 points and the line l be drawn so that none of the 5 points lie on l ? Can they be drawn so that only 1 of them lies on l , etc.
12. Let A, B, C, D and E denote the vertices of a regular pentagon in the plane. If a line is drawn through each pair of these points, into how many regions is the plane divided?
13. Six consecutive integers are written on a blackboard. When one of them is erased, the sum of the remaining five is 1999. What number was erased?

14. An octagon is inscribed in a circle. If the lengths of the eight sides are 2, 3, 2, 3, 2, 3, 2 and 3, in that order, find the area of the octagon in the form $a + b\sqrt{c}$ where a , b , and c are integers.

