

EF Exam

1. Find the point on the circle of radius 4 centered at $(3, 4)$ which is closest to the point $(8, 16)$.
2. Suppose that a particle sitting at the point $(3, 6)$ is rotated $\pi/6$ radians in the clockwise direction about the point $(-1, 2)$. Give the coordinates of the new location of the particle.
3. Suppose you enter a chain mail scheme with the following instructions:

** There are 3 names and addresses listed in order below. Place a \$5 bill in an envelope and mail it to the first person on the list. Then remove the first person from the list and move the second and third persons to the first and second slots. Finally, add your name and address to the third slot on the list and mail these instructions along with the names and addresses to 20 people.

If every person who receives this mailing follows the directions perfectly, then what is the amount of money you will receive?

4. A particle leaves the point $(0, 4)$ in the direction of the point $(100, 54)$ with constant speed of 1 unit per second. A second particle leaves the point $(-1, 1)$ in the direction of the point $(201, 203)$ with some unknown constant speed. If the two particles collide then what is the speed of the second particle?
5. Compute the following limit:

$$\lim_{h \rightarrow 0} \frac{\sin(1) \cos(h) + \cos(1) \sin(h) - \sin(1)}{h}$$

6. Suppose $a \neq 0$ and $b > 3a^2$. If a particle travels from the point (a, b) towards the point $(a, 3a^2)$ and reflects off of the parabola $y = 3x^2$, then where will it cross the y -axis?

7. Suppose a computer program generates random matrices of the form

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

where a, b, c, d are integers chosen at random from the set $\{-3, -2, -1, 0, 1, 2, 3\}$. What is the probability that such a matrix would have a zero determinant? That is, what is the probability that $ad - bc = 0$?

8. Suppose k and m are positive integers and $x_1 = 1$. For $n = 1, 2, 3, \dots$ define

$$x_{n+1} = x_n - \frac{1}{k} \left(x_n - \frac{m}{x_n^{k-1}} \right)$$

Find $\lim_{n \rightarrow \infty} x_n$.

9. A positive integer x is chosen with $10 \leq x \leq 99$. If x is divided by the sum of its digits, how small can the result be?
10. Suppose that two vertical poles with heights 10 feet and 40 feet are connected with one wire from the top of the 40 foot pole to the base of the 10 foot pole, and a second wire from the top of the 10 foot pole to the base of the 40 foot pole. What is the height of the crossing point of these two wires?
11. An octagon is inscribed in a circle. The lengths of consecutive sides of the octagon alternate between 2 inches and 3 inches. Find the area of the octagon in the form $a + b\sqrt{c}$ where a, b and c are integers.
12. Compute the following limit:

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \sin \left(\frac{k\pi}{n} \right).$$

13. Suppose an infinite matrix is constructed with decimal numbers as shown below

$$\begin{pmatrix} 1.1 & 1.2 & 1.3 & 1.4 & \cdots \\ 2.1 & 2.2 & 2.3 & 2.4 & \cdots \\ 3.1 & 3.2 & 3.3 & 3.4 & \cdots \\ 4.1 & 4.2 & 4.3 & 4.4 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

and then the entries are written in a sequence as follows:

$$\{1.1, 2.1, 1.2, 3.1, 2.2, 1.3, 4.1, 3.2, 2.3, 1.4, 5.1, \dots\}$$

Note that the first entry in this sequence is 1.1, the second entry is 2.1, and so on. Which entry in the sequence is 1171.2831?

14. Each of 117 crates in a supermarket contains at least 80 and at most 102 apples. What is the least number of crates that must contain the same number of apples?
15. Consider the equation $3x + 4y = 188$. What integral solution (x, y) yields the least positive difference $y - x$?
16. What is the largest positive integer that must be a factor of $n^5 - 5n^3 + 4n$ for all positive integers n ?
17. Find the constant b_3 so that the polynomial

$$f(x) = 2 - 3x + 4x^2 - 6x^3 + 12x^4 - x^5$$

can be written in the form

$$f(x) = b_0 + b_1(x - 1) + b_2(x - 1)^2 + b_3(x - 1)^3 + b_4(x - 1)^4 + b_5(x - 1)^5$$