

“Wavelet Sets in N-Dimensional Space”

by

Krista Rister, Texas A&M University

While wavelet sets on the real number line may sometimes be difficult to grasp visually, and thus require an accompanied algebraic explanation, wavelet sets in higher dimensions such as \mathbb{R}^2 have a geometric representation that makes them somewhat easier to grasp and much more interesting to look at. Understanding them, however, does involve several of the same basic principles required to comprehend wavelet sets in one dimension. Ideas such as wavelets and dilation and translation congruence can be extended into the higher dimensions. For example, if a set in \mathbb{R}^2 is 2δ -translation congruent to the square $[-\delta, \delta) \times [-\delta, \delta)$ and 2-dilation congruent to the square torus $[-\delta, \delta) \times [-\delta, \delta) \setminus [-\delta/2, \delta/2) \times [-\delta/2, \delta/2)$, then it is a wavelet set with dilation matrix $2I$. This rule allows for the construction of sample wavelet sets, and samples constructed under these guidelines this summer will be shared. However, different dilation matrices yield wavelet sets that look far different from those with dilation matrix $2I$. Examples will be shown. Another interesting idea is the extension of a known wavelet set into a higher dimension. Is there an analogous set in \mathbb{R}^n ? Although it is not known for the general case, specific cases can be studied. There are also many theoretical questions about multiple dimensional wavelet sets. Many of these questions seem to have obvious answers, but proving them is far more difficult than it would seem.