Texas A&M University:Mathematics Dept.
REU/MCTP/UBM
Undergraduate Research Conference: July 27-28, 2011

Wednesday July 27
9:20-9:30 Al Boggess and Sue Geller: Opening Remarks

Session 1 Moderator: May Boggess
9:30-9:50 Eradicating Invasive Species through Sex Reversal,
Katie Storey (Carleton College)
Dr. May Boggess, Dr. Jay Walton, and Dr. Xueying Wang.
An invasive species causes harm to the habitat in which it resides, and the Trojan Y Chromosome Model is a potential method for eradicating invasive species of fish. Feminized YY Supermales of the targeted invasive species are added to an ecosystem, causing the female XX fish to decrease in population until they reach a specified level. If the level of female fish is low enough when the addition of feminized supermales stops, the species will die out. An ordinary differential equation model, a stochastic model, and a spatial model were used to determine the level of female fish necessary to ensure the eradication of the species.

9:55-10:15 A Mathematical Model of Chagas Disease,
Sara Krueger (Bethany Lutheran College)
Dr. May Boggess and Dr. Jay Walton
More than 1 billion people all over the world are infected with neglected tropical diseases, such as Chagas disease in Latin and South America. Like malaria, this disease is passed between infected animals and people by an insect, the reduviid bug. In this paper, an ordinary differential equations model was developed that accounts for the existence of a disease reservoir in mammals, such as domestic livestock. Our results show that the disease free state is not a stable equilibrium, meaning that the introduction of even a small number of insects will lead to an epidemic.

10:20-10:40 Bounding the Number of Components of Polynomial Hypersurfaces,
Daniel Smith (UC Berkeley)
Dr. J. Maurice Rojas
Finding real roots of polynomials is an almost everyday task in many settings. Determining all real roots, however, is far from simple, especially when considering polynomials in multiple variables. Often having bounds on some characteristic of the zero set of a polynomial helps us determine if we have found all real roots. Recent advances in research have refined bounds on the number of connected components of positive real zero sets of polynomials. I will discuss these bounds and in so doing demonstrate techniques that allow us to determine exact bounds, where they exist. In particular I will be discussing Viro diagrams and discriminant amoebae. I shall also show a new bound on the number of compact connected components of 2-variate 5-nomials.

10:45-11:05 Amoebas and their Tropical Varieties
Timothy Jewell (University of California, Los Angeles)
Dr. J. Maurice Rojas
Amoebas make obvious extraordinarily beautiful art simply within the polynomials of several variables. They allow for a unique perspective of the zero set, and have many

remarkable properties. In particular, a recent result of Martin Avendao expands our understanding of these constructions through tropical varieties, allowing us to observe the relationship between the Archimedean-Newton Polygons, tropical varieties and amoebas of polynomials. In particular, these provide an observation linking our logarithmic zero set with the geometric interpretation of a polynomial of several variables.

Session 2 Moderator: Jay Walton

11:30-11:50 Approximating A-Discriminant Amoebae of Tetranomials,
Katherine Turner (Texas A&M University)
Dr. J. Maurice Rojas

In characterizing the zero sets of polynomials, a useful criterion to know is if they lie outside of their respective discriminants. Amoebae are particularly advantageous in showing the location of these polynomials as their outer chambers hold more information about the zero sets. We explore a new approximation of the body of tetranomial $\mathcal{A}$-discriminant amoebae through the Horn-Kapranov parametrization method. We take linear combinations of the parametrization's derivatives and find the points of tangency to create a polygon that, while an underestimation of the amoeba body, can be used to visualize the location of a polynomial with respect to the discriminant.

11:55-12:15 Implementing Nilsson and Passare's Coamoeba Algorithm,
Jeff Sommars (Wheaton College)
Dr. J. Maurice Rojas

Coamoebae were defined for the first time within the past ten years and since then, much fruitful research has been done on them. Two years ago, Lisa Nilsson and Mikael Passare wrote a paper that carefully describes an algorithm for drawing specific two dimensional coamoeba. I developed a program in Sage that takes an A matrix as input from a user and returns the corresponding coamoeba, with several options for ways the user can adapt the coamoeba graph returned. I will present the general ideas of their paper, and will conclude by showing a number of A matrices and their respective coamoebas. This program has the potential to aid future mathematical research that seeks to gain new insight into coamoebas.

12:20-12:40 Mathematical Modeling of Circannual Prolactin Cycles,
Paulo Eusebio (Washburn University)
Dr. Jay Walton

In this presentation, we examine how the daily melatonin signals of Soay sheep affects their circannual prolactin secretion by means of time delay and negative feedback. The author proposes a nonlinear system of delay differential equations to model the system using the same data as previous ones but with a more simple, reduced conceptual model.

12:45-1:05 Feasibility of p-adic polynomial systems,
Davi da Silva (University of Chicago)
Dr. J. Maurice Rojas

The p-adic number system is pertinent to many fields, including cryptography, and many of these applications naturally rely on solving systems of polynomials over the p-adics. The question of whether, in general, such a polynomial system has a root over $\mathbb{Q}_p$ – and whether this can be verified algorithmically – is therefore of practical and theoretical importance. Some general problems in the search for p-adic polynomial roots are discussed, as are some results on the existence and computability of p-adic roots.

Session 3 Moderator: Sue Geller

2:00-2:20 Matrix Analysis of Lone Star Tick Populations,
Adam Kley (Texas A & M University)
Dr. Bill Grant, Dr. Hsiao-Hsuan (Rose) Wang, Dr. May Boggess

Lone Star ticks are important to model because they carry harmful bacteria. In this project we implemented the 1987 matrix model of Haile and Mount, which accounts for the affects of changes in relative humidity, daylength, and temperature on survival, reproduction, and growth rates. Our results show that certain life stages are more sensitive to climatic changes than others, and that the most sensitive life stages are not constant year round.

2:25-2:45 Fraction of Nonnegative Polynomials that are Sums of Squares,
Caitlin Lownes (Massachusetts Institute of Technology)
Dr. J. Maurice Rojas

Polynomials that are sums of squares (SOS) can be efficiently optimized via semidefinite programming. In this paper, we investigate when a nonnegative polynomial $p \in \mathbb{R}[x_1, \ldots, x_n]$ *is SOS. For nonnegative polynomials of fixed degree, previous results by Blekherman show that, as* $n \to \infty$*, the fraction of nonnegative polynomials that are SOS approaches zero. However, these bounds are loose, and this fraction is unknown for most polynomials of low degree in few variables. Our research focuses on estimating this fraction for nonnegative bivariate polynomials of degree at most four in each variable. The fraction of nonnegative polynomials that are SOS can be estimated as the ratio of volumes of two naturally definable convex bodies of dimension 24. To avoid computing these volumes directly, we implemented a version of Smith's rapidly mixing hit and run technique for uniform sampling from a convex body.*

2:50-3:10 Frame Theory Over Arbitrary Fields,
Suren Jayasuriya (University of Pittsburgh),
Pedro Perez (Columbus State University)
Dr. David Larson

The study of frames for vector spaces has become of great importance over the past three decades, forming the theoretical basis behind applications in signal processing and sampling theory. In 2009, Bodman, et al. began to investigate frames over $\mathbb{Z}_2$ *in "Frame Theory for Binary Vector Spaces". Motivated by their work, we develop frame theory for finite-dimensional vector spaces over arbitrary fields* $\mathbb{F}$ *that may have a degenerate bilinear form. To overcome the degeneracy of the bilinear form, we introduce the characterization of an analysis frame as a frame for a vector space such that the analysis operator* $\Theta : V \to \mathbb{F}^k$ *defined by* $\Theta(x) = (\langle x, x_1 \rangle, \langle x, x_2 \rangle, \ldots, \langle x, x_k \rangle)^T$ *is injective. We establish equivalent results on vector spaces that admit an analysis frame, called analysis spaces, including a reconstruction formula, Riesz Representation theorem, and existence of a dual frame pair. Defining an subspace of* V *as an zero inner product space* $ZIP(V) := \{x \in V | \langle x, y \rangle = 0 \ \forall y \in V\}$*, we prove that every finite-dimensional vector space over an arbitrary field can be written as the algebraic direct sum of an analysis space and its zero inner product space.*

3:15-3:35 Ray sensory neuron control of the C. elegans nematode worm mating behavior,
Laura Caflisch (Texas A & M University)
Dr. Robyn Lints and Dr. May Boggess

Many evolutionarily significant behaviors, such as mating, involve dynamic interactions with animate targets. This raises the question of what features of neural circuit design are essential to support these complex types of behavior. The C. elegans male uses 18 ray sensilla of the tail to induce and coordinate a systematic search for the hermaphrodite vulva, but precisely how ray neuron types, A and B, endow the male with a high degree of spatial and temporal precision is unknown. By specifically killing either A- or B-neuron populations, we show that both neuron types are necessary for search initiation but only A-neurons are necessary for later phases of the search. Ray neurons control search initiation and trajectory by inducing ventral bending of the tail. This bending can be induced in the absence of a mating partner by artificially activating the neurons using specialized molecular tools. We have quantified the postures produced by measuring the angles of various points along the males body axis. Calculation and comparison of angles produced by A- and B-neuron populations shows that, although the postures produced are superficially similar, there are statistically significant differences between them. Together these data reveal that the robustness of the search stem from considerable functional overlap between ray neuron populations (i.e. all induce ventral bending) and precision is due to the subtle differences in their output (the degree of ventral bending is identical).

4:00-4:20 Fractals By Hand
May Boggess

Graduate School Overview
4:20-4:40 Paulo Lima-Filo: TAMU Mathematics Department
4:40-5:00 Fred Dahm: TAMU Statistics Department

Thursday July 28

Session 1 Moderator: Dave Larson

9:30-9:50 *λ-permutations of Conditionally Divergent Series I,*
Jacob Brumbaugh-Smith (Pomona College)
Dr. David Larson, Dr. Yunus Zeytuncu, and Stephen Rowe

t $\lambda - permutations$ are rearrangements (permutations) of the natural numbers which preserve convergence of all *convergent series and which make at least one divergent series convergent. We can consider how λ-permutations act on divergent series in general. Fix a divergent series $\sum a_n$ and consider the set S of all possible limits of convergent λ-permutations of $\sum a_n$. We have examples of series where $S = \mathbb{R}$ or $S = \emptyset$, and have not been able to construct series with other behavior. An open problem, due to Velleman is to* prove or disprove: "$S = \mathbb{R}$ or $S = \emptyset$" *. In this talk we will develop the main tool at our disposal: a necessary and sufficient condition for a permutation to be convergence preserving. We will then use this characterization to calculate the S sets of two different series.*

9:55-10:15 *λ-permutations of Conditionally Divergent Series II,*
Mamikon Gulian (University of Maryland, Baltimore County)
Dr. David Larson, Dr. Yunus Zeytuncu, and Stephen Rowe

This talk continues to describe the partial progress towards solving the Velleman problem for λ-permutations as described in Jacob Brumbaugh-Smith's talk. I will present a method based on performing surgery on λ-permutations of series, which can be loosely described as "shifting blocks", to perturb the limits of series to any given value. We prove that the modified permutations are still λ-permutations for certain classes of series, thus solving Velleman's problem for many special cases. However, problems arise for series which develop self-similiar oscillations, where the modified permutatations can have unbounded block number. This gives insight into what properties a counterexample must have.

10:20-10:40 *Implications of Harvest Strategies in Fishery Populations,*
Corinne Wentworth (St. Mary's College of Maryland)
Dr. Masami Fujiwara and Dr. Jay Walton

Fishery management is the consideration of the ecological effects of harvesting. Fisherman work to provide fish for a growing human population but because of this some fish populations have been dangerously declining. It is important to balance ecological and economic needs. In this paper we investigate various deterministic models of fishery populations. A simple logistic model, a skewed logistic model with a square term, and a model that demonstrates the Allee effect have all been considered with a constant harvest rate as well as time dependent havesting. Optimization and numerical calculations were used to calculate the harvest rate that produces maximum yield under different population density scenerios.

10:45-11:05 *Carotid Artery Model Using Criscione Kinematic Framework,*
Carlos Grahm (Texas A & M University)
Dr. John Criscione and Dr. Jay Walton

The material properties of arteries are poorly understood. These properties are important in explaining pathology of vessels under a variety of conditions. Previous attempts of carotid models have been based on ill-conceived frameworks which are not suited to solve extension and inflation, one of the simplest problems involving tubes. The Criscione kinematic framework minimizes covariance and simplifies balance equations. The framework provides basis for a model which is invertible and well suited to model arterial behavior.

Session 2 Moderator: Maurice Rojas

11:30-11:50 Finger Motion Modeling For Bionic Fingers,

Myrielle Allen-Prince (Bennett College)

Dr. Jay Walton

The use of bionic hands are becoming a reality for those who have suffered amputation. In this project we modeled the motion of the finger as it bends in and out using Newton's third law of motion. A system of partial differential equations was developed to describe the relationship of the forces needed to move the finger to a specified position, which incorporated a feedback mechanism. Our work shows that this type of model can be used to accurately control the motion of a human finger.

11:55-12:15 On Chord Recognition,

Zach Gendreau (Worcester Polytechnic Institute), Westin King (Baylor University),

Derek Kinney (University of Rochester), Mallory Schulte (Augustana College),

Kelly O'Bryant (Texas A&M University at Corpus Christi)

Gregory Berkolaiko, Constanze Liaw, and Joe Bartley

We constructed algorithms to distinguish various guitar chords using wavelet decomposition and the Discrete Fourier transform. The data collected and utilized consists of recordings of B minor, E minor, F minor, E major, E major 9, and E minor 7. Due to the sensitive nature of the data, it is necessary to closely analyze each note, clear dead space, and select four similar repetitions of each note. The algorithm created will recognize the difference between the notes by computing the Frobenius norm.

12:20-12:40 Methods of Cursive Letter Recognition,

Josh Bracewell (New Jersey Institute of Technology),

Aviva Bukiet (New York University), Amanda Hoisington (College of the Sequoias),

Kermit Martignoni (Southern Illinois University Edwardsville), Mandy Welch (Northern Kentucky University)

Dr. Gregory Berkolaiko, Dr. Constanze Liaw, and Joseph Bartley

We present methods of recognizing lowercase cursive letters. We discuss wavelet decomposition, Fourier analysis, and additional techniques that we developed in order to identify the various characteristics of cursive letters.

12:45-1:05 Refinability of Multivariate Polynomials,

Paul Gustafson (Princeton University)

David Larson

We completely characterize the polynomials in several variables that are refinable by a scalar dilation. We also consider matrix dilations and obtain results for some special classes of matrix dilations. There are some open questions remainingt that we discuss.

Session 3 Moderator: Greg Berkoliko

2:00-2:20 Oregon Blackberry Invasion Analyzed by Spatial Stochastic Modeling,

Heather Johnston (Western Oregon University)

Dr. May Boggess and Dr. Jay Walton

Nonnative species are disruptive to natural ecosystems. The Himalayan Blackberry in Western Oregon is of particular concern due to the massive barriers the plant creates and its stubbornness of eradication. A spatially stochastic model is used to predict the spread of the blackberry in Oregon, taking into account factors such as elevation, annual rainfall, temperature, urban sprawl and black bear ranges.

2:25-2:45 *Word Recognition,*

M. Harding Jr. (Morehouse College),

S. Lane (College of St. Benedict & St. John's University),

E. Metheney (SIU Edwardsville), K. Stoutz (State University of New York at Geneseo),

R. Whitaker (Carleton College)

Joe Bartley, Gregory Berkolaiko, and Constanze Liaw

Instilled in our everyday lives, the technicalities of word recognition can easily be foreseen. We can see word recognition and its importance through military fighter aircrafts, helicopters, and telephone directories. From using these devices the following question arises: How do the machines recognize words? We constructed two algorithms to distinguish several words we recorded. One uses wavelets and the other, Fourier transform.

2:50-3:10 *Signature Recognition Using Wavelets and Other Methods,*

Katelyn Heath (St. Michael's College), Kiley Johnson (Seattle Pacific University),

Bryan Karlovitz (Westchester University),

Joshua Vaughan (Mississippi State University)

Molly Stubblefield (Western Oregon University),

Dr. Gregory Berkolaiko and Dr. Constanze Liaw

In this presentation, we will utilize Fourier transform, Wavelet decomposition, and other similar methods to discover algorithms for identifying forged signatures. In addition, we will look at more sophisticated methods used by professionals for signature verification and recognition.

3:15-3:35 *Uncertainty and Information in Time-Frequency Analysis,*

Suren Jayasuriya (University of Pittsburgh)

Dr. David Larson and Dr. Lewis Bowen

We discuss uncertainty principles in time-frequency analysis and their connection with central ideas of information theory which were introduced by C.E. Shannon in his 1948 paper, "A Mathematical Theory of Communication". In 1957, I. Hirschman proved an uncertainty result that related the entropies of a function and its Fourier transform. Similar to Donoho and Stark's generalization of the classical Fourier uncertainty principle that relies on the approximate concentration of Fourier transform pairs on their supports, we formulate a new concept of approximate entropic concentration of random variables associated with the Fourier transform pair $x, \hat{x} \in \mathbb{R}^N$*. We prove that the sum of the approximate entropies is bounded below by* $\log_2(N) - \delta$ *where* δ *is related to the approximation error. This talk will be self-contained and presupposes no specialized background in either time-frequency analysis or information theory.*

4:00 **Awards Ceremony and Closing Remarks.** *May Boggess.*