Inoculation Strategies for Polio:
Modeling the Effects of a Growing Population on Public Health Outcomes

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Inoculation Strategies for Polio:
Model

Figure: Basic SIR Model
Figure: Basic SIR Model with Non-Constant Population
Figure: Basic SIR Model with Non-Constant Population and Death from Disease
Epidemiology

Epidemic
A rapid spread, growth, or development.

Endemic
Maintained in a population without external inputs.
Epidemiology

Epidemic
A rapid spread, growth, or development.

Endemic
Maintained in a population without external inputs.
Epidemic

When is $I'(t) > 0$?

$$I'(t) = \beta S \frac{I}{N} - \delta I - dI - \gamma I > 0$$

$$R_0 := \frac{\beta}{\delta + d + \gamma} > 1$$

$$R_{0_{\text{constant}}} := \frac{\beta}{\gamma} > 1$$
Epidemiology

Epidemic

When is $I'(t) > 0$?

\[ I'(t) = \beta S \frac{I}{N} - \delta I - dI - \gamma I > 0 \]

\[ R_0 := \frac{\beta}{\delta + d + \gamma} > 1 \]

\[ R_{0constant} := \frac{\beta}{\gamma} > 1 \]
Epidemiology

Epidemic
When is $I'(t) > 0$?

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$$R_0 := \frac{\beta}{\delta + d + \gamma} > 1$$

$$R_{0\text{constant}} := \frac{\beta}{\gamma} > 1$$
**Epidemic**

When is \( I'(t) > 0? \)

\[
l'(t) = \beta S \frac{I}{N} - \delta I - dI - \gamma I > 0
\]

\[
R_0 := \frac{\beta}{\delta + d + \gamma} > 1
\]

\[
R_{0\text{constant}} := \frac{\beta}{\gamma} > 1
\]
Endemic

What is the end behavior of $I(t)$?

\[
\frac{d \left( \frac{I}{N} \right)}{d \left( \frac{S}{N} \right)} = \frac{(I/N)'(t)}{(S/N)'(t)} > 0
\]
Endemic
What is the end behavior of \( I(t) \)?

\[
\frac{d \left( \frac{I}{N} \right)}{d \left( \frac{S}{N} \right)} = \frac{(\frac{I}{N})'(t)}{(\frac{S}{N})'(t)} > 0
\]
Figure: Basic SIR Model with Non-Constant Population, Death from Disease, and Vaccination of Newborns
Model: Vaccination

\[ (1 - \alpha) \rho N \]

\[ \beta S \frac{I}{N} \]

\[ \delta S \]

\[ \delta I \]

\[ \gamma I \]

\[ \delta R \]

\[ \alpha \rho N \]

**Figure**: Basic SIR Model with Non-Constant Population, Death from Disease, and Vaccination of Newborns
Evaluating the Disease Free State

\[S(t) = (1 - \alpha) \delta S\]

\[R(t) = \alpha \delta R\]

\[I(t) = 0\]

\[N(t)\]

\[(1 - \alpha) \rho N\]
Evaluating the Disease Free State

\[
\begin{align*}
\delta S &= (1 - \alpha)\rho N \\
\delta R &= \alpha\rho N
\end{align*}
\]

\[
\frac{S}{N}(t) = (1 - \alpha) \\
\frac{I}{N}(t) = 0 \\
\frac{R}{N}(t) = \alpha
\]
Preventing an Epidemic

When is $I'(t) < 0$?

$$R_0 := \frac{\beta S}{\delta + d + \gamma} < 1$$

$$\frac{\beta (1 - \alpha)}{\delta + d + \gamma} < 1$$

$$1 - \frac{\delta + d + \gamma}{\beta} < \alpha$$
Model: Split Age Classes

\[
\begin{align*}
\delta_C S_C &= \rho N_A \\
\beta S_C l_C + l_A &= l_C \\
\gamma_C l_C &= R_C \\
\delta_C R_C &= \theta R_C \\
\beta S_A l_C + l_A &= l_A \\
\gamma_A (1 - \mu) l_A &= R_A \\
\delta_A R_A &= \delta_A S_A \\
\theta S_C &= \delta A S_A \\
\gamma_C l_C &= \gamma_A (1 - \mu) l_A \\
\delta_C S_C &= \delta A R_A
\end{align*}
\]

Figure: Split Age Class SIR Model
Model: Vaccinating 1-4 Year-Olds

\[
\begin{align*}
\delta_C S_C & \quad \beta S_C \frac{l_C + l_A}{N} \\
S_C & \quad \beta S_A \frac{l_C + l_A}{N} \\
\theta S_C & \quad \theta S_C \\
\delta_A S_A & \quad \delta_A S_A \\
\gamma_C l_C & \quad \gamma_A (1 - \mu) l_A \\
l_C & \quad l_C \\
\gamma_C l_C & \quad \gamma_C l_C \\
R_C & \quad R_C \\
\delta_C R_C & \quad \delta_C R_C \\
\rho N_A & \quad \rho N_A \\
\delta_C I_C & \quad \delta_C I_C \\
\theta R_C & \quad \theta R_C \\
\delta_A R_A & \quad \delta_A R_A \\
\end{align*}
\]

Figure: Split Age Class SIR Model For 1-5 Year-Old Vaccination Strategy
Model: Vaccinating 1-4 Year-Olds

Figure: Split Age Class SIR Model For 1-5 Year-Old Vaccination Strategy
Model: Vaccinating 1-4 Year-Olds

\[ R_0 := \frac{\beta S_C(t)}{N(t)} + \frac{\beta S_A(t)}{N(t)} < 1 \]

\[ \beta = \text{infectious rate} = 0.60 \]

\[ \delta_C = \text{child mortality rate} = 0.124 \]

\[ \delta_A = \text{adult mortality rate} = 0.013 \]

\[ \gamma_C = \text{maturation rate} = 0.049 \]

\[ \gamma_A = \text{maturation rate} = 0.25 \]

\[ \rho = \text{birth rate} = 0.038 \]

\[ \theta > 0.0095 \]
Model: Vaccinating 1-4 Year-Olds

\[ R_0 := \frac{\beta \frac{S_C(t)}{N(t)}}{\delta_C + \gamma_C} + \frac{\beta \frac{S_A(t)}{N(t)}}{d\mu + \gamma_A(1 - \mu)} < 1 \]

\[ \delta_A(\delta_C + \theta) > \rho \theta \]
Model: Vaccinating 1-4 Year-Olds

\[ R_0 := \frac{\beta S_C(t)}{\delta_C} + \frac{\beta S_A(t)}{\delta_A} < 1 \]

\[ \delta_A (\delta_C + \theta) > \rho \theta \]

\[ \rho = \text{birth rate} = 0.038 \]
\[ \delta_C = \text{child mortality rate} = 0.124 \]
\[ \theta = \text{maturation rate} = 0.25 \]
\[ \delta_A = \text{adult mortality rate} = 0.013 \]
Model: Vaccinating 1-4 Year-Olds

\[ R_0 := \frac{\beta \frac{S_C(t)}{N(t)}}{\delta_C + \gamma_C} + \frac{\beta \frac{S_A(t)}{N(t)}}{d\mu + \gamma_A(1 - \mu)} < 1 \]

\[ \delta_A(\delta_C + \theta) > \rho \theta \]

\( \rho = \text{birth rate} = 0.038 \)

\( \theta = \text{maturation rate} = 0.25 \)

\( \delta_C = \text{child mortality rate} = 0.124 \)

\( \delta_A = \text{adult mortality rate} = 0.013 \)

\( 0.0049 \not< 0.0095 \)
Model: Vaccinating Newborns (Age 0)

\[ \delta_C S_C \quad \beta S_C \frac{l_C + l_A}{N} \quad \gamma_C l_C \quad \delta_C R_C \]

\[ \rho N_A \quad \theta S_C \quad \beta S_A \frac{l_C + l_A}{N} \quad \gamma_A (1 - \mu) l_A \quad \delta_A R_A \]

**Figure:** SIR Model For Newborn Vaccination Strategy

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Model: Vaccinating Newborns (Age 0)

\[(1 - \alpha) \rho N_A\]

\[\delta_C S_C\]

\[\beta S_C \frac{l_C + l_A}{N}\]

\[\delta_C l_C\]

\[\gamma_C l_C\]

\[\delta_C R_C\]

\[\alpha \rho N_A\]

\[\theta S_C\]

\[\theta R_C\]

\[\delta_A S_A\]

\[\beta S_A \frac{l_C + l_A}{N}\]

\[\delta_A R_A\]

\[\gamma_A (1 - \mu) l_A\]

\[\delta_A S_A\]

\[d\mu l_A\]

\[\delta_A R_A\]

Figure: SIR Model For Newborn Vaccination Strategy
Model: Vaccinating Newborns (Age 0)

\[ R_0 := \frac{\beta S_C(t)}{\delta_C + \gamma_C} + \frac{\beta S_A(t)}{N(t)} < 1 \]
Model: Vaccinating Newborns (Age 0)

\[ R_0 := \frac{\beta S_C(t)}{N(t)} + \frac{\beta S_A(t)}{N(t)} < 1 \]

\[ 1 - \frac{\delta_A (\delta_C + \theta)}{\rho \theta} < 2 \alpha \]
Model: Vaccinating Newborns (Age 0)

\[ R_0 := \frac{\beta S_C(t)}{\delta_C + \gamma_C} + \frac{\beta S_A(t)}{d\mu + \gamma_A(1 - \mu)} < 1 \]

\[ 1 - \frac{\delta_A(\delta_C + \theta)}{\rho\theta} < 2\alpha \]

\[ \rho = \text{birth rate} = 0.038 \]
\[ \theta = \text{maturation rate} = 0.25 \]

\[ \delta_C = \text{child mortality rate} = 0.124 \]
\[ \delta_A = \text{adult mortality rate} = 0.013 \]

\[ 0.244 < \alpha \]
Model: Vaccinating Newborns (Age 0)

\[ R_0 := \frac{\beta S_C(t)}{N(t)} \frac{S_C(t)}{N(t)} + \frac{\beta S_C(t)}{N(t)} \frac{S_A(t)}{N(t)} < 1 \]

\[ 1 - \frac{\delta_A(\delta_C + \theta)}{\rho \theta} < 2\alpha \]

\[ \rho = \text{birth rate} = 0.038 \]
\[ \delta_C = \text{child mortality rate} = 0.124 \]
\[ \theta = \text{maturation rate} = 0.25 \]
\[ \delta_A = \text{adult mortality rate} = 0.013 \]
\[ 0.244 < \alpha \]
Minimum $\alpha = 0.78$
Results

Minimum $\alpha = 0.78$

$\alpha = 0.6$

$\alpha = 0.9$
Future Work

- Combination model for vaccination
- Continued analysis of age structures
- Proof of equilibrium state taking into consideration death from disease
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