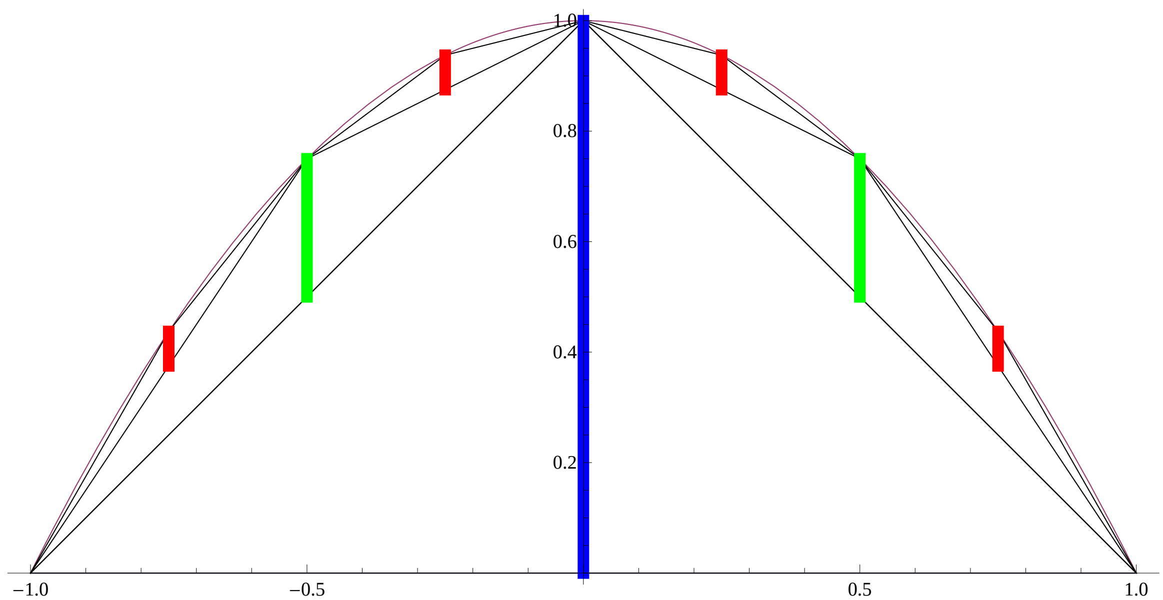
Geometric series through time

Archimedes and the parabola.

Archimedes and the area enclosed by a parabola and chord.



Archimedes got the area, here 4/3, by the following analysis. The first triangle has a certain area, readily calculated. The next two triangles, the green ones, each have height 1/4 the blue one, and base half the blue one, so each green-height triangle has 1/8 the area of the blue-height triangle. But then, there are two of them. The greens give 1/4 the area of the blue. In like manner, the reds give area 1/4 the greens, and so on. Geometric series, posit a value X for the total, and observe that X=1+X/4. Thus X=4/3.

Of course, there's a lot of geometry under the hood here. Nowadays, we use a lot of algebra in our geometry, and things would go like this:

The height of a triangle nestled into a chord going from (a,1-a^2) to (b,1-b^2), with third vertex at ((a+b)/2,(1-((a+b)/2)^2), is the vertical distance between (1-(a+b)^2/4) and (1-(a^2+b^2)/2), and algebraically, that works out to (a^2+b^2)/2-(a+b)^2/4=(b-a)^2/4.

So the height is proportional to the square of the horizontal span of the chord. Thus, halving the chord quarters the height and multiplies the area by 1/8.

Under the hood: (click on menu to open closed cells).

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pix1 = Plot[{0, 1 - x^2}, {x, -1, 1}, AspectRatio -> Automatic]
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