How shall we find the volume got by revolving the region between the line y=x and the curve y=x^2 about the line y=x? The axis of rotation does not lie along the x or y axis or parallel to it, so we shall have to extract the relation between r and s from the formula for the curve. Now s is the distance along the line y=x from the origin to where we take a right angle and go a distance r out to the point (x,y). The line joining (x,y) to (y,x) cuts our axis of rotation at the point ((x+y)/2,(x+y)/2) which lies at a distance of (x+y)/sqrt(2) from the origin. So, s=(x+y)/sqrt(2). The radius r is the distance from that point to (x,y) which works out to r=(x-y)/sqrt(2). Now we fire up Maple and get it to solve our pair of equations and express x and y in terms of r and s, instead of the other way around like we now have it. This doesn't require Maple; in principle it's just like solving say r=2*x+3*y,s=5*x+7*y for x and y in terms of r and s. That's a bit of a pain, and our actual equation is a bit more of a pain because of the square roots in it. Anyhow, here's what Maple gets:

```
> solve([s=(x+y)/sqrt(2),r=(x-y)/sqrt(2)],[x,y]);
```

[[x = 1/2*2^(1/2)*(s+r), y = 1/2*2^(1/2)*(-r+s)]]

Now we want to convert our formula y=x^2 into a formula relating r and s: What would be just as good would be to convert the expression y-x^2 into an expression in terms of r and s. Here it is:

```
>
> ?subs
> subs({x = 1/2*2^(1/2)*(s+r), y = 1/2*2^(1/2)*(-r+s)},y-x^2);
```

1/2*2^(1/2)*(-r+s)-1/2*(s+r)^2

Now we want to express r as a function of s alone, so we can integrate. Here we go:

```
> solve(%=0,r);
```

-1/2*2^(1/2)-s+1/2*(2+8*2^(1/2)*s)^(1/2), -1/2*2^(1/2)-s-1/2*(2+8*2^(1/2)*s)^(1/2)

Finally, we integrate Pi*(r(s))^2 for s from 0 to sqrt(2), to get our volume.

```
> Pi*int((-1/2*2^(1/2)-s+1/2*(2+8*2^(1/2)*s)^(1/2))^2,s=0..sqrt(2));
```

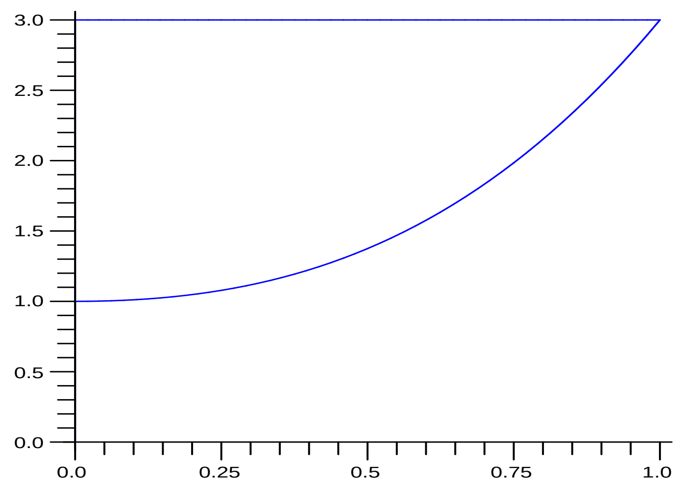
1/60*Pi*2^(1/2)

Well, it's easier when the axis of rotation runs parallel to either the x or y axis. But if it doesn't, the problem may yet be workable.

```
>
```

New topic: cylindrical shells and disks give the same answer. Example: rotate the region bounded above by the line y=3, below by the curve y=x^3+x^2+1, and on the left by the y axis, about that y axis.

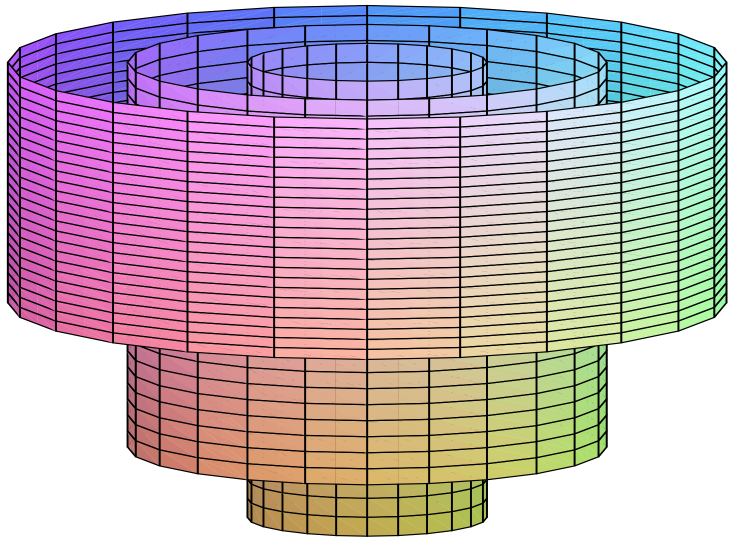
```
>
>
> plot({[t,t^3+t^2+1,t=0..1],[t,3,t=0..1],[0,t,t=0..3]},color=[blue,blue,blue]);
```



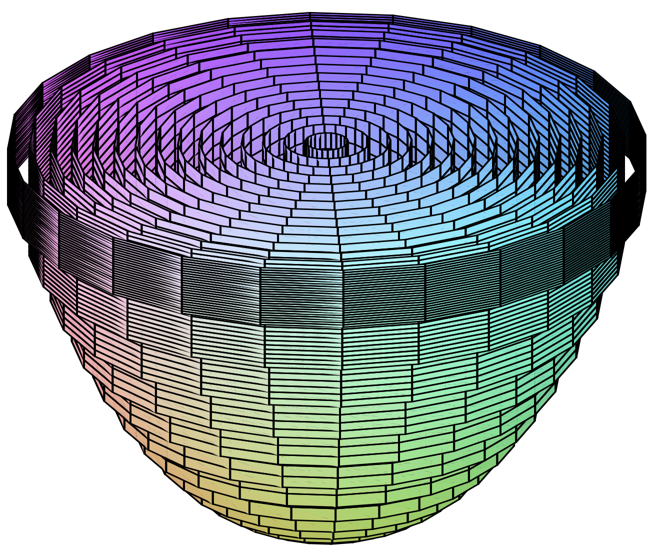

```
> with(plots);
```

[Interactive, animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, cylinderplot, densityplot, display, display3d, fieldplot, fieldplot3d, gradplot, gradplot3d, graphplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, replot, rootlocus, semilogplot, setoptions, setoptions3d, spacecurve, sparsematrixplot, sphereplot, surfdata, textplot, textplot3d, tubeplot]

```
> ?cylinderplot
> c1:=cylinderplot(1/2,theta=0..2*Pi,z=11/8..3):
> c2:=cylinderplot(3/4,theta=0..2*Pi,z=127/64..3):
> c0:=cylinderplot(1/4,theta=0..2*Pi,z=69/64..3):
> ?display
> display({c0,c1,c2});
```



```
>
>
> cc:=[seq(cylinderplot(k/16,theta=0..2*Pi,z=(k/16)^3+(k/16)^2+1..3),k=1..15)]:
> display(cc);
```



```
>
>
>
>
>
```

```
>
>
>
>
>
```

The two integrals below are what arise out of the disks and cylindrical shells approach to the volume of our region based on the curve x cubed plus x squared plus 1. The first one comes from the disk method, which is on the face of it unworkable because it is too difficult to solve for x in terms of y to get the r that goes with our disk method. But after doing that substitution we talked about in class, with the implicit function, you get this first integral. The second integral results from a straightforward cylindrical shells calculation.

```
> int(x^2*(3*x^2+2*x),x=0..1);
```

11/10

```
> int(2*x*(3-(x^3+x^2+1)),x=0..1);
```

11/10

```
>
>
>
>
>
>
>
```