

Experiments, Sample Spaces and Events

An EXPERIMENT is any activity with an observable result. Tossing a coin, rolling a die or choosing a card are all considered experiments.

An OUTCOME (or SAMPLE POINT) is the result of a the experiment. The set of all possible outcomes or sample points of an experiment is called the SAMPLE SPACE.

An EVENT is a subset of the sample space.

Example - A coin is tossed and the side noted and a card is drawn and the color noted.

a) What is the sample space?

b) What events are possible?

Example - From a bin of 20 apples a sample of 4 is chosen. The number of rotten apples in the sample is counted. What is the sample space?

Definition of Probability

To find the theoretical probability of an event occurring we must first find a UNIFORM SAMPLE SPACE (the outcomes are all equally likely). If there are n outcomes in the sample space, they will each have a probability of $1/n$ of occurring. The outcomes are MUTUALLY EXCLUSIVE - that is, only one can occur during the experiment.

We want to arrange the outcomes (also called simple events) in a probability distribution table:

outcome	probability
$\{s_1\}$	$1/n$
$\{s_1\}$	$1/n$
\vdots	\vdots
$\{s_1\}$	$1/n$

We say $P(\{s_1\}) = P(s_1) = P_1 = 1/n$

Probability distribution tables have the following properties:

$$1. 0 \leq P(s_i) \leq 1$$

$$2. P(s_1) + P(s_2) + \dots + P(s_n) = 1$$

$$3. P(\{s_i \cup s_j\}) = P(s_i) + P(s_j), i \neq j$$

Example - A card is chosen from a standard 52 card deck and the suit is noted. Find the probability distribution table for this experiment:

outcome	probability
♥	1/4
♠	1/4
♣	1/4
♦	1/4

What is the probability that the card is red (♥ OR ♦)?

$$P(\text{red}) = 1/4 + 1/4 = 2/4$$

The empirical probability of an outcome is determined by the relative frequency it occurs. You can find the relative frequency by doing an experiment.

Example - A survey was done of students for how many earrings they are wearing. The following results were found:

no. of earrings	no. of students	relative frequency
0	100	$100/345 = .29$
1	60	$60/345 = .17$
2	120	$120/345 = .35$
3	30	$30/345 = .09$
4	20	$20/345 = .06$
5 or more	15	$15/345 = .04$
total	345	1

So we can find the probability that a person has 3 or more earrings on as

$$P(x > 3) = P(3) + P(4) + P(5 \text{ or more})$$

$$= 30/345 + 20/345 + 15/345 = .19$$

Rules of Probability

We have seen when the events are mutually exclusive that the probability of both events occurring is the sum of the two probabilities. If E and F are mutually exclusive then

$$P(E \cup F) = P(E) + P(F)$$

Now let's consider events that have some outcomes in common. We will use the union rule for probability:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Example - consider rolling two fair die, one red and one green. The sample space has 36 equally likely outcomes. What is the probability of rolling a sum of 5 or a 1 on the red die?

$$E = \{x | x \text{ is a sum of } 5 \}$$

$$= \{1 - 4, 2 - 3, 3 - 2, 4 - 1\}$$

$$F = \{x | x \text{ is a } 1 \text{ on the red die} \}$$

$$= \{1 - 1, 2 - 1, 3 - 1, 4 - 1, 5 - 1, 6 - 1\}$$

$$E \cap F = \{4 - 1\}$$

$$P(E) = 4/36, P(F) = 6/36, P(E \cap F) = 1/36.$$

$$P(E \cup F) = 4/36 + 6/36 - 1/36 = 9/36$$

Sometimes you need to use a Venn diagram to help sort things out:

Let E and F be two events with a sample space S .

Suppose that $P(E) = .7$, $P(F) = .5$

and $P(E \cup F) = .9$.

Find $P(E \cap F)$, $P(E \cup F^c)$, $P(E^c \cap F^c)$.

Draw a Venn diagram...