Math142 Lecture Notes
4.3 - Derivatives of Exponential Functions

Derivative Rules for Exponential Functions

- The derivative of $f(x) = e^x$ is $f'(x) = e^x$
- If f is a differentiable function of x, the derivative of $h(x) = e^{f(x)}$ is $h'(x) = e^{f(x)}(f'(x))$
- Given $y = e^{(\mathbf{mess})}$ where **mess** represents some function in x, the derivative is given by $y' = e^{(\mathbf{mess})} \cdot \mathbf{mess'}$
- If f is a differentiable function of x, and $b > 0, b \neq 1$, the derivative of $h(x) = b^{f(x)}$ is $h'(x) = b^{f(x)}(\ln b)(f'(x))$
- Given $y = b^{\mathbf{mess}}$ where **mess** represents some function in x, the derivative is given by $y' = b^{\mathbf{mess}} \cdot \mathbf{mess'} \cdot \ln\ b$

Example 1: Differentiate and simplify (unless otherwise stated) the following.

(a) $g(x) = 5e^x$

(b) $g(x) = 6x^3 \cdot e^x$

(c) $g(x) = e^{4x+5}$

(d) $g(x) = (3x^2)(e^{4x^3})$

(e) $g(x) = 8^x$

(f) $g(x) = \dfrac{8^x \ln(x^2)}{2^x}$ (do not simplify)

(g) $g(x) = (\log_3\ 9x)(5^{x-e})$ (do not simplify)

(h) $g(x) = (e^{3x^2+2x})^3 2^{\ln(x)}$ (do not simplify)

(i) $g(x) = \dfrac{(\log_3 x^2)e^{\sqrt{x}}}{(\ln(2x))^3}$ (do not simplify)

Example 2: The number of civil cases going to trial in U.S. district courts can be modeled by

$$f(x) = 12.4(0.95)^x, \quad 1 \leq x \leq 12$$

where x represents the number of years since 1984 and $f(x)$ represents the number of thousands of civil cases going to trial in U.S. district courts.[1]

(a) Classify the function as exponential growth or decay model. Explain.

(b) Determine $f'(x)$ for the model. Evaluate and interpret $f'(3)$.

(c) Rewrite the model in the form $g(x) = a \cdot e^{kx}$, where a and k are constants rounded to the nearest hundredth.

[1] *Brief Calculus*: Armstrong & Davis, p. 248, # 52