Assignment # 3
Date due: Tuesday, May 3, 2005.

Remark: This assignment concerns a geometric system \([S, \mathcal{L}, \mathcal{P}, d, m]\) consisting of a space \(S\), containing a set of lines \(\mathcal{L}\), a set of planes \(\mathcal{P}\), a distance function \(d\) and an angular measure function \(m\). The system is subject to the Incidence Postulates, Distance Postulates, Plane Separation and Space Separation postulates.

Problem 1 (20pts). Given a set \(A\) in space, the convex hull of \(A\) is the intersection of all convex sets that contain \(A\). Show that the convex hull of \(A\) is convex.

Problem 2 (20pts). If \(L\) is a line in plane \(H\), and \(H_1\) is a side of the line in \(H\), show that \(H_1 \equiv L\) is convex.

Problem 3 (20pts). Given a triangle \(\triangle ABC\), let \(L\) be a line in the same plane of the triangle. Show that if \(L\) contains no vertex of the triangle, then \(L\) cannot intersect all of the three sides.

Problem 4 (20pts). Let \(L\) be a line in the same plane of a triangle \(\triangle ABC\). Show that if the line intersects the interior of the triangle, then it must intersect at least one of the sides.

Problem 5 (20pts). Show that if the diagonals of a quadrilateral intersect, then the quadrilateral is convex. (Please read carefully the definition of convex quadrilateral.)

Problem 6 (20pts). Show that the lines containing the diagonals of a quadrilateral intersect, even if the quadrilateral is not convex.

Problem 7 (20pts). Prove that if two intersecting lines form one right angle, then they form 4 right angles. Please state clearly all postulates and results used. (This is the statement of Theorem 6, p. 99, in the book. You cannot quote this theorem, of course.)

Problem 8. Show that if two angles of a triangle are congruent then the sides opposite them are also congruent.