Math 142 Week-in-Review #3 (Exam 1 Review: Sections A.8, 1.0-1.3, 1.5, and 3.1-3.3)

Note: This collection of questions is intended to be a brief overview of the exam material (with emphasis on sections 3.2 and 3.3). When studying, you should also rework your notes, the previous week-in-reviews for this material, as well as your suggested and online homework.

1. Find \( \lim_{x \to -1} \frac{x^2 + 4x + 3}{(x + 1)^2} \), if it exists. If it does not exist, use limits to describe the way in which it does not exist.

2. The population of a small city in Texas for various years is shown in the table below. (Midyear estimates are given.)

<table>
<thead>
<tr>
<th>Year</th>
<th>2003</th>
<th>2005</th>
<th>2007</th>
<th>2009</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>25,416</td>
<td>30,307</td>
<td>35,206</td>
<td>40,105</td>
<td>44,996</td>
</tr>
</tbody>
</table>

   a) Find the average rate of growth from 2005 to 2009, and interpret your answer. Round to the nearest integer if necessary.

   b) Estimate the instantaneous rate of growth during 2007, and interpret your answer. Round to the nearest integer if necessary.
3. Given the function \( f(x) \) below,

\[
f(x) = \begin{cases} 
\sqrt{x^2 - \pi} & x < -5 \\
\log_7(x + 9) & -5 \leq x < 2 \\
\frac{x + 2}{x^2 - x - 6} & -5 \leq x < 2 \\
e^{\sqrt{x}} & x > 2 
\end{cases}
\]

a) find the domain of \( f(x) \).

b) determine where \( f(x) \) is continuous.

c) find \( f(9) \), if it exists.

d) find \( \lim_{x \to 2} f(x) \), if it exists.

e) find \( f(2) \), if it exists.

f) find \( \lim_{x \to -2} f(x) \), if it exists.
4. Use calculus to determine where the function \( f(x) = x^2 - 3x + 15 \) has a horizontal tangent line.

5. Solve the following for \( x \):
   
   a) \( \log_7 \left( \log_5 (2x + 1) + 45 \right) = \log_5 25 \)
   
   b) \( \log_7 (3 - x) = \log_7 54 - \log_7 (-x) \)
6. The owners of a local candy shop have determined that if they charge $20 for a gift basket, they can sell 100 baskets each month. If they increase the price by $25, they only sell 75 gift baskets. The owners have monthly fixed costs of $1,500, and it costs them $40 to make each basket. They have decided they will not supply any gift baskets if the price is $50 or less, but for each increase in price of $1, they will supply an additional basket. Answer the following and round to the nearest integer if necessary.

   a) How many gift baskets do they need to sell to break-even, assuming they sell at least 32 baskets?

   b) Find the equilibrium price.

   c) Find the average rate of change in revenue when production increases from 35 baskets to 40 baskets each month, and interpret your answer.
d) Find the rate of change in profit at a production level of 35 baskets, and interpret your answer.

e) What price should they charge to maximize revenue?

7. Suppose the effective yield for an investment that compounds continuously is 6.72%. Find the nominal (annual) interest rate.
8. Sketch the derivative of $f(x)$ shown below.

![Diagram of a function and its derivative]

9. Use the definition of the derivative to find $f'(x)$ if $f(x) = \sqrt{5x-1}$, and then use your formula to find the equation of the line tangent to $f(x)$ at $x = 2$. 

![Equation of a tangent line]
10. For the function $f$ whose graph is shown below, find each of the following.

a) State the domain of $f$.

b) Where is $f$ continuous?

c) Show (using the definition of continuity) that $f$ is continuous at $x = -5$.

d) For what values of $c$ does $\lim_{x \to c} f(x)$ not exist? For each value of $c$, use limits to describe the way in which the limit does not exist.

e) Which condition in the definition of continuity fails first at $x = 6$?

f) For what values of $a$ does $f'(a)$ not exist? For each value of $a$, explain why the derivative does not exist.
11. Sketch the graph of a function \( f \) that is continuous everywhere, but it is not differentiable at \( x = -4 \) because there is no tangent line or at \( x = 3 \) because the tangent line is vertical.

12. Given \( a \) is a constant and \( a > 0 \), determine where \( f(x) = \frac{4 \cdot 3^{2x-5}}{\log_7(a-x)} \) is continuous.

13. 200 grams of a pollutant were buried. After 3 years, there were 84 grams left. Assuming continuous decay, how many grams of the pollutant will be left after 4.2 years?
14. If an object is launched into the air with a velocity of 50 feet per second, its height \( t \) seconds later is given by \( s(t) = -16t^2 + 50t \) feet.

a) Find the average velocity between two and three seconds.

b) Find the velocity of the object after two seconds.

15. Classify each of the following as a power function, rational function, polynomial function (state its degree and leading coefficient), or none of these (state why).

a) \( f(x) = \frac{x^2 - 7}{3x - \pi} \)

b) \( g(x) = 2x^6 - 4x^4 - \sqrt{x^2} + 10x - 9 \)
16. If \( f(x) = \frac{(x-a)(x-b)}{|x-a|} \), find \( \lim_{x \to a^-} f(x) \).

17. Solve for \( x \):
   
   a) \( e^{2x}x^2 - e^{2x}4x = 5e^{2x} \)

   b) \( 27^x - 534 = \log_8 8 \)

18. Indicate verbally how the graph of \( g(x) = -3e^{x+4} + 8 \) is related to the graph of \( f(x) = e^x \).