Math 142 Week-in-Review #4 (Sections 4.1, 4.2, and 4.3)

1. If \( j(x) = \frac{2e^x}{5} - \frac{\sqrt{x^3}}{x^2} + \ln \frac{5}{x^2} - \pi x - 1.8^3 \), find \( \frac{dj}{dx} \).

2. \( \frac{d}{dx} \left( \frac{x^{-8} + 3x^2 - \sqrt[7]{x^3}}{8 \sqrt{x^5}} + 3^x \right) \)

3. Find the equation of the line tangent to the curve \( y = \frac{x^2 - 3x - 4e^x}{x - 5} \) at \( x = 0 \).
4. Find the derivative of each of the following functions. Do not simplify your answers.

a) \( f(x) = \sqrt{x} \left( 2x^2 - 4x + 7 \right) \left( \frac{4}{x^9} + 2(8^x) - 3 \log_6 x \right) \)

b) \( f(x) = \frac{(x^3 - 7x + \pi^2)e^x}{3\sqrt[3]{x^3} - x^4 + 2x - 3} \)

c) \( f(x) = (4 \ln x) \sqrt{\sqrt{2x^2 + 3x + 4} - 5e^x} \)
d) \( f(x) = \sqrt[5]{\frac{3x^3 - 4x^2 + 2\pi x}{\log_2 x - \frac{2}{3}x^4}} \)

e) \( f(x) = \frac{(2\log x) \left(3x^2 - 5x + 4\right)^5}{\sqrt{x + 5}} \)

5. Given \( k(p) = \frac{5}{p^2} \) and \( p(h) = 1 - (6h^4 + h)^2 \), find \( \frac{dk}{dh} \).
6. The total cost (in hundreds of dollars) of producing \( x \) cameras per week is 
\[ C(x) = 6 + \sqrt{4x+4}, \]
where \( 0 \leq x \leq 30 \).

a) Find \( C(24) \), and interpret your result.

b) Find the marginal cost when 24 cameras are produced, and interpret your result.

c) Estimate total cost when 25 cameras are produced.

d) Find the exact cost when 25 cameras are produced.

e) Approximate the cost of the 16th camera.

f) Find the exact cost of the 16th camera.

g) Find \( C(20) \), and interpret your result.
7. Use the information in the table below regarding the functions $f(x)$ and $g(x)$ to answer questions a) - e).

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
<th>$g(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>30</td>
<td>12</td>
<td>24</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>-6</td>
<td>0</td>
<td>0</td>
<td>2</td>
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<td>10</td>
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<td>4090</td>
<td>128</td>
<td>4224</td>
<td>130</td>
</tr>
</tbody>
</table>

(a) If $m(x) = \frac{f(x)}{g(x)}$, find $m'(5)$.

(b) If $n(x) = 2f(x)g(x)$, find $n'(0)$.

(c) If $h(x) = x^2 - 3(g(x))^4$, find $h'(5)$. 
d) If \( j(x) = f(x)g(x^2) \), find \( j'(8) \).

e) If \( k(x) = f(f(g(x))) \), find \( k'(0) \).

8. The position of a particle is given by \( s = t^3 - 10.5t^2 + 30t, \ t \geq 0 \), where \( t \) is time in seconds and \( s \) is measured in meters.

a) Find the velocity after 4 seconds.

b) When is the particle at rest?