Math 142 Week-in-Review #4 (Sections 4.1, 4.2, and 4.3)

1. If \( j(x) = \frac{2e^x}{5} - \sqrt{x^3 + \ln \frac{5}{x}} - \pi x - 1.8^3 \), find \( \frac{dj}{dx} \).

   \[ j(x) = -\frac{3}{5} e^x - x^{3/8} + \ln 5 - 2\ln x - \pi x - 1.8^3 \]

   \[ \frac{dj}{dx} = -\frac{3}{5} e^x - \frac{3}{8} x^{-5/8} - 2 \left( \frac{1}{x} \right) - \pi \]

2. \( \frac{d}{dx} \left( \frac{x^{8/3} + 3x^2 - \sqrt{x^3}}{8x^{1/3}} + 3^x \right) \)

   \( \text{One term or rewrite (or use quotient rule)} \)

   \[ \frac{d}{dx} \left( \frac{x^{8/3} + 3x^2 - \sqrt{x^3}}{8x^{1/3}} + 3^x \right) = \frac{d}{dx} \left( \frac{1}{8} x^{8/3} + \frac{3}{8} x^{4/3} - \frac{1}{8} x^{1/3} + 3^x \right) \]

   \[ = \frac{2x^{5/3}}{24} x^{5/3} + \frac{12}{24} x^{1/3} x^{1/3} - \frac{1}{12} x^{-5/12} + 3^x \ln 3 \]

3. Find the equation of the line tangent to the curve \( y = \frac{x^2 - 3x - 4e^x}{x - 5} \) at \( x = 0 \).

   \[ \text{slope} \Rightarrow y' = \frac{(x-5)(2x-3-4e^x) - (x^2-3x-4e^x)(1)}{(x-5)^2} \]

   \[ y'(0) = \frac{(-5)(-3-4e^0) - (-4e^0)(1)}{(-5)^2} = \frac{35 + 4}{25} = \frac{39}{25} = m \]

   \[ \text{point} \Rightarrow x=0 \text{ then } y = f(0) = \frac{0 - 0 - 4e^0}{0 - 5} = \frac{-4}{-5} = \frac{4}{5} \Rightarrow (0, \frac{4}{5}) \]

   \[ y - \frac{4}{5} = \frac{39}{25} (x - 0) \Rightarrow \frac{y}{39/25} = \frac{39}{25} x + \frac{4}{5} \]
4. Find the derivative of each of the following functions. Do not simplify your answers.

a) \( f(x) = \sqrt{2x^2 - 4x + 7} \left( \frac{4}{x^5} + 2(8^x) - 3 \log_5 x \right) \)  
\( \frac{d}{dx} \left( \frac{4}{x^5} + 2(8^x) - 3 \log_5 x \right) = \frac{ FS' + SF' }{ \text{(Rewrite)} } \)

\[ f(x) = (2\sqrt{2} \cdot \frac{5}{2} - 4 \cdot \frac{3}{2} + 7 \cdot \frac{1}{2}) \left( 4x^{-9} + 2(8^x) - 3 \log_5 x \right) \]

\[ f'(x) = \left( 2x^{\frac{5}{2}} - 4 \cdot x^{\frac{3}{2}} + 7 \cdot x^{\frac{1}{2}} \right) \left[ -36x^{-10} + 2(8^x \ln 8) - 3 \left( \frac{1}{5} \cdot \frac{1}{x} \right) \right] + \left( 4x^{-9} + 2(8^x) - 3 \log_5 x \right) \left[ \frac{10}{2} \cdot x^{\frac{3}{2}} - \frac{12}{3} \cdot x^{\frac{1}{2}} + \frac{7}{2} \cdot x^{-\frac{1}{2}} \right] \]

b) \( f(x) = \frac{(x^3 - 7x + \pi)^e}{3x^5 - x^4 + 2x - 3} \)  
\( \frac{d}{dx} \left( \frac{3x^5 - x^4 + 2x - 3}{x^3 - 7x + \pi} \right) \)  
\( \text{quotient rule then product rule for top} \)

\[ f'(x) = \left( (3x^{\frac{3}{5}} - x^\frac{1}{4} + 2x^{-3}) \left[ (x^3 - 7x + \pi^2) e^x + e^x (3x^2 - 7) \right] \right) \left( (x^3 - 7x + \pi^2) e^x \right) \left[ \left( \frac{1}{5} x^{-\frac{1}{5}} - 4x^3 + 2 \right) \right] \]

\[ \left( \frac{1}{3} x^{\frac{3}{5}} - x^{\frac{1}{4}} + 2x^{-3} \right)^2 \]

c) \( f(x) = \sqrt{2x^2 + 3x + 4 - 5e^x} \)  
\( \text{product rule and chain rule} \)

\[ f'(x) = (4 \ln x) \left[ \frac{1}{2} \left( (2x^2 + 3x + 4)^{\frac{1}{2}} - 5e^x \right)^{\frac{1}{2}} \left[ \frac{1}{2} (2x^2 + 3x + 4)^{\frac{1}{2}} (4x + 3) - 5e^x \right] \right] + \left( (2x^2 + 3x + 4)^{\frac{1}{2}} - 5e^x \right)^{\frac{1}{2}} \left[ 4 \left( \frac{1}{x} \right) \right] \]
d) $f(x) = \sqrt{\left(\frac{3x^3 - 4x^2 + 2\pi x}{\log_2 x - \frac{3}{4}x^4}\right)^5}$  
* Chain rule then quotient rule.

\[
\begin{align*}
F'(x) &= \frac{5}{6} \left(\frac{3x^3 - 4x^2 + 2\pi x}{\log_2 x - \frac{3}{4}x^4}\right)^{-\frac{1}{6}} \\
&= \frac{(\log_2 x - \frac{3}{4}x^4)^{\frac{5}{6}}}{(3x^3 - 4x^2 + 2\pi x)} - \frac{\partial}{\partial x} \left[\left(\frac{1}{\log_2 x}\right)\right] - \frac{\partial}{\partial x} \left[\left(\frac{3}{8}x^3\right)\right]
\end{align*}
\]

\[
= \frac{(\log_2 x - \frac{3}{4}x^4)^{\frac{5}{6}}}{(3x^3 - 4x^2 + 2\pi x)} - 3x^2 - 8x + 2\pi
\]

\[
= \frac{(\log_2 x - \frac{3}{4}x^4)^{\frac{5}{6}}}{(3x^3 - 4x^2 + 2\pi x)} - \frac{3}{8}x^3
\]

c) $f(x) = \frac{(2\log x)(3x^2 - 5x + 4)^5}{\sqrt{x+5}}$  
* Quotient, product, chain rules.

\[
F'(x) = \left[(x+5)^{-\frac{1}{2}} \left[\frac{\partial}{\partial x} \left(2\log x\right)\left[5(3x^2 - 5x + 4)^4(6x - 5)\right] + (3x^2 - 5x + 4)^5 \left[\frac{1}{x}\right]\right]\right] / \left[(x+5)^{-\frac{1}{2}}\right]
\]

\[
= \frac{2\log x (3x^2 - 5x + 4)^5 \left[\frac{1}{2} (x+5)^{-\frac{1}{2}} (6x - 5)\right]}{\left[(x+5)^{-\frac{1}{2}}\right]^2}
\]

5. Given $k(p) = \frac{5}{p^2}$ and $p(h) = 1 - (6h^4 + h)^2$, find \( \frac{dk}{dh} \) variable in answer is \( h \)

\[
\frac{dk}{dh} = \frac{dk}{dp} \cdot \frac{dp}{dh}
\]

\[
\Rightarrow -10p^{-3} \cdot (-2(6h^4 + h)(24h^3 + 1))
\]

\[
\Rightarrow \text{sub. } u \text{ for } p \Rightarrow \frac{dk}{dh} = -10(1 - (6h^4 + h)^2)^{-3} \cdot (-2(6h^4 + h)(24h^3 + 1))
\]
6. The total cost (in hundreds of dollars) of producing \( x \) cameras per week is \( C(x) = 6 + \sqrt{4x+4} \), where \( 0 \leq x \leq 30 \).

a) Find \( C(24) \), and interpret your result.

\[
C(24) = 16 \text{ hundred dollars,} \quad \text{when 24 cameras are made, total cost is 16 hundred dollars, (or $1,600)}
\]

b) Find the marginal cost when 24 cameras are produced, and interpret your result.

\[
C'(x) = \frac{1}{2} \cdot \frac{4}{4x+4} = \frac{1}{2} \cdot 4 = 2 \quad \Rightarrow \quad C'(24) = 0.20 \text{ hundred dollars per camera, (or $0.20 per camera)}
\]

When 24 cameras are made, total cost is increasing by 0.20 hundred dollars per camera.

(c) Estimate total cost when 25 cameras are produced.

\[
C(25) \approx C(24) + C'(24) = 16 + 2 = 16.2 \text{ hundred dollars, (or $1,620)}
\]

d) Find the exact cost when 25 cameras are produced.

\[
C(25) = 16.1980 \text{ hundred dollars, (or $1,619.80)}
\]

e) Approximate the cost of the 16th camera.

\[
C'(15) = 0.25 \text{ hundred dollars, (or $0.25)}
\]

f) Find the exact cost of the 16th camera.

\[
C(16) - C(15) = 24.62 \text{ hundred dollars, (or $24.62)}
\]

g) Find \( \bar{C}(20) \), and interpret your result.

\[
\bar{C}(x) = \frac{C(x)}{x} \quad \text{(average cost)}
\]

\[
= \frac{6 + \sqrt{4x+4}}{x} \quad \Rightarrow \quad \bar{C}(20) = 0.7583 \text{ hundred dollars, (or $75.83)}
\]

When 20 cameras are made, the average cost per camera is 0.7583 hundred dollars.
7. Use the information in the table below regarding the functions \( f(x) \) and \( g(x) \) to answer questions a) - e).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-6</th>
<th>0</th>
<th>5</th>
<th>8</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>30</td>
<td>-6</td>
<td>19</td>
<td>58</td>
<td>4090</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>-12</td>
<td>0</td>
<td>10</td>
<td>16</td>
<td>128</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>24</td>
<td>0</td>
<td>35</td>
<td>80</td>
<td>4224</td>
</tr>
<tr>
<td>( g'(x) )</td>
<td>-10</td>
<td>2</td>
<td>12</td>
<td>18</td>
<td>130</td>
</tr>
</tbody>
</table>

a) If \( m(x) = \frac{f(x)}{g(x)} \), find \( m'(5) \).

\[
m'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \Rightarrow m'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2}
\]

\[
= \frac{35(10) - 19(13)}{(35)^2}
\]

\[
= \frac{350 - 247}{1225}
\]

\[
= \frac{103}{1225}
\]

b) If \( n(x) = 2f(x)g(x) \), find \( n'(0) \).

\[
n'(x) = 2[f(x)g'(x) + g(x)f'(x)] \Rightarrow n'(0) = 2[f(0)g'(0) + g(0)f'(0)]
\]

\[
= 2[-6(2) + 0(13)]
\]

\[
= -24
\]

c) If \( h(x) = x^2 - 3(g(x))^4 \), find \( h'(5) \).

\[
h'(x) = 2x - 12(g(x))^3g'(x)
\]

\[
h'(5) = 2(5) - 12(g(5))^3g'(5)
\]

\[
= 10 - 12(35)^3(12)
\]

\[
= -6,173,990
\]
d) If \( f(x) = g(x^2), \) find \( j'(8) \).

\[
j'(8) = f(8) \left[ g'(16) \left[ \frac{16}{2} \right] \right] + g'(x^2) f'(x)
\]

\[
= 58 \left[ 130 \cdot 16 \right] + 4224 (16)
\]

\[
= \boxed{188,224}
\]

e) If \( k(x) = f(f(x)) \), find \( k'(0) \).

\[
k'(0) = f'(f(0)) f'(g(0)) g'(0)
\]

\[
= f'(-6) f'(0) (2)
\]

\[
= (-12) (0) (2) = 0
\]

8. The position of a particle is given by \( s = t^3 - 10.5t^2 + 30t, \ t \geq 0 \), where \( t \) is time in seconds and \( s \) is measured in meters.

a) Find the velocity after 4 seconds.

\[
V(t) = S'(t) = 3t^2 - 21t + 30 \quad \Rightarrow \quad V(4) = \boxed{-6 \text{ m/s}}
\]

b) When is the particle at rest?

When \( V(t) = 0 \) \( \Rightarrow \)

\[
3t^2 - 21t + 30 = 0
\]

\[
3 (t^2 - 7t + 10) = 0
\]

\[
3 (t - 5)(t - 2) = 0
\]

\[
t = \boxed{5 \text{ sec.}}
\]

\[
t = \boxed{2 \text{ sec.}}
\]