1. Find the derivative of each of the following functions. Do not simplify your answers.

a) \( f(x) = \sqrt[4]{\ln(7x^2 - 5)} \left[ \log(x^3 + 9^x) \right] \)

\[
f'(x) = \frac{1}{4} \ln(7x^2 - 5) \left[ \frac{1}{x} \left( \frac{1}{x^2 + 9} \right)(3x^2 + 9^x \ln 9) \right] + \\
\left[ \log(x^3 + 9^x) \right] \left[ \frac{3}{4} \left( \frac{1}{7x^2 - 5} \right)(14x) \right]
\]

b) \( f(x) = \frac{\log_8(6x^2 + \pi)}{7(e^x - 3)} \)

\[
\left( \frac{e^x - 3}{7(e^x - 3)} \right) \left[ \frac{1}{6x^2 + \pi} \left( 12x \right) \right] - \left( \log_8(6x^2 + \pi) \right) \left[ 7e^x \ln(7)(e^x) \right] / \left( 7(e^x - 3) \right)^2
\]
c) \( f(x) = \sqrt[3]{(e^{3x^2} \ln x)^2} \)  

\[ f'(x) = \frac{2}{3} \left( e^{3x^2} \ln x \right)^{-\frac{1}{3}} \]

\[
\left[ e^{3x^2} \left( \frac{1}{x} \right) + (\ln x) [e^{3x^2} (6x)] \right]
\]

2. List the three conditions necessary for a function \( f(x) \) to have a relative maximum or minimum at \( x = c \).

1. \( f' = 0 \) or \( f' \) DNE.
2. \( f \) is defined at \( x = c \). (i.e. in domain of \( f \))
3. \( f' \) changes sign at \( x = c \).
3. A local coffee shop has a weekly price demand equation of \( p = 30 - \frac{c}{20} \), where \( p \) is the price (in dollars) for a cup of coffee and \( x \) is the number of cups sold.

a) If the current price for a cup of coffee is $5.00, describe the effect on demand if price increases by 12%. Is demand elastic, inelastic, or unit elastic at the current price level? (change)

\[ a_0 p = 600 - x \]
\[ x = 600 - a_0 p \]
\[ \Rightarrow E(p) = \frac{-p f'(p)}{f(p)} = \frac{-p (-20)}{600 - a_0 p} \]
\[ E(s) = 0.2 \quad < 1 \Rightarrow \text{inelastic} \]
\[ \Rightarrow 0.2 \times (18\%) = 3.6 \% \Rightarrow \text{demand down by 3.6\%} \]

\% change demand = \( E(p) \times \% \text{change price} \)

b) At what price level is the shop's revenue maximized?

\[ \frac{20p}{600 - 20p} = 1 \quad \Rightarrow \quad 20p = 600 - 20p \]
\[ 40p = 600 \]
\[ p = \frac{15}{2} \]

\[ p = 7.5 \]

\[ e(p) = 1 \]

\[ E(p) = \frac{20p}{600 - 20p} > 1 \quad \Rightarrow \quad 20p > 600 - 20p \]
\[ 40p > 600 \]
\[ p > 15 \]

\[ \times \text{ need upper bound for } p! \]

\[ x > 0 \Rightarrow 600 - 20p > 0 \Rightarrow 600 > 20p \]
\[ \Rightarrow p < 30 \]

\[ (15, 30) \]
4. For each of the following functions, find any critical values, the intervals where the function is increasing/decreasing, and any relative extrema. Be sure to classify the type of each extrema.

a) \( f(x) = x - \ln(x) \)

1. Domain: \( x > 0 \) \( \Rightarrow (0, \infty) \)
2. Partition #’s \( f' \):
   - \( f' = 1 - \frac{1}{x} \)
   - \( x = 1 \)
3. Critical values of \( f' \):
   - \( x = 1 \)

   Sign chart of \( f' \):
   - \( f' : \bigcirc \bigcirc \bigcirc \)
   - \( f' \downarrow \uparrow \)

   Den. on \((0,1)\)
   Dom. on \((1, \infty)\)
   Local \text{min} \ at \(x = 1\)
   \( f(1) = 1 \)

b) \( f(x) = \frac{2xe^x}{e^x - 1} \)

1. Domain: \( (-\infty, \infty) \)
2. Partition #’s \( f' \):
   - \( f' = x^2e^{-x}(1) + e^x(\Delta x) \)
   - \( x^2e^{-x}(-x + 2) \)
   - \( x = 0 \)
   - \( e^{-x} = 0 \)
   - \( -x + 2 = 0 \)
   - \( x = 2 \)
3. Critical values of \( f' \):
   - \( x = 0 \)
   - \( x = 2 \)

   Sign chart of \( f' \):
   - \( f' : \bigcirc \bigcirc \bigcirc \)
   - \( f' \downarrow \uparrow \)

   Den. on \((-\infty, 0)\) and \((2, \infty)\)
   Local \text{rel min} \ at \(x = 0\)
   is \( f(0) = 0 \)
   Local \text{rel max} \ at \(x = 2\)
   is \( f(2) = \frac{4}{e^2} \)
c) \( f(x) = \frac{3x^2 - 2x}{(x-4)^2} \)

1. Domain: \((-\infty, 4) \cup (4, \infty)\)

2. Partition #1's for \( f' \):
   \[
   f' = \frac{(x-4)^2 [6x - 2] - (3x^2 - 2x)2(x-4)(1)}{(x-4)^2} \]
   \[= \frac{(x-4)[(6x-2) - (3x^2 - 2x)]}{(x-4)^2} \]
   \[= \frac{(6x^2 - 2x^2 - 4x + 8)}{(x-4)^3} \]
   \[= \frac{2x^2 - 4x + 8}{(x-4)^3} \]

3. Critical values of \( f \):
   \[ x = \frac{8}{22} \]

4. Sign chart for \( f' \):
   
   \[
   f' \left\{ \begin{array}{c}
   - & 0 & + & - \\
   \frac{8}{22} & + & 4 & - \\
   f & \downarrow & \uparrow & \downarrow
   \end{array} \right. 
   \]

   - Min. on \((\frac{8}{22}, 4)\)
   - Dec. on \((-\infty, \frac{8}{22})\) and \((4, \infty)\)

   Local min at \( x = \frac{8}{22} \)
   
   \[ f(\frac{8}{22}) = \ldots \]

Title: Feb 25-10:20 AM (5 of 10)
5. Suppose a company that makes deluxe toasters has a weekly demand equation given by \( p(x) = 150e^{-0.02x} \), where \( p \) is the price in dollars when \( x \) toasters are sold. Determine where the company's revenue is increasing.

\[ R = x \cdot p = 150xe^{-0.02x} \]

1. Domain: \([0, \infty)\) *because \(x \geq 0\) for # of toasters

2. Partition #’s of \( R'\):

\[ R'(x) = 150x \left[ e^{-0.02x} \cdot (-0.02) \right] + e^{-0.02x} \left[ 150 \right] \]

\[ = 150 e^{-0.02x} \left( -0.02x + 1 \right) \]

\[ R' = 0 \Rightarrow -0.02x + 1 = 0 \Rightarrow x = 50 \]

R’ DNE \( \Rightarrow \) Never!

3. Critical values of \( R\): \( x = 50 \)

4. Sign chart of \( R'\):

\[ R' \begin{cases} + & 0 < x < 50 \\ - & x > 50 \end{cases} \]

\( \text{Dnn. m} (0, 50) \)

\( \text{always parenth.} \)
6. Briefly state the difference between partition numbers and critical values of a function.

Partition numbers are where \( f' = 0 \) or \( f' \text{ does not exist} \). Critical values are also where \( f' = 0 \) or \( f' \text{ does not exist} \), but they also have to be in the domain (i.e., \( f \) is defined).

Critical values are a subset of partition numbers. They are partition numbers that are in the domain of \( f \).

7. Doc and Marty’s Skateboard Shop has determined the price-demand equation for its “futuristic” skateboards to be \( x + 0.10p = 37.5 \), where \( x \) is the number of skateboards that can be sold at a unit price of \( p \) dollars. The current price for a skateboard is $150. In order to increase their revenue, what should the Doc and Marty do?

\[
\text{solve for } x = f(p) ! \quad x = 37.5 - 0.10p
\]

\[
E(p) = \frac{-p(-0.10)}{37.5 - 0.10p} \Rightarrow E(150) = 0.3 < 1
\]

\( \Rightarrow \text{inelastic} \)

\( \text{max. revenue } \Rightarrow \text{increase price} \)
8. Consider a function $f$ that is smooth and continuous on its domain of $(-\infty, 2) \cup (2, \infty)$.
Also, $f'(x) = \frac{8(x+4)}{(x-2)^4}$. Find any critical values of $f(x)$, the intervals where $f(x)$ is increasing/decreasing, as well as where any relative extrema occur (be sure to specify the type of any relative extrema).

1. Domain: $(-\infty, 2) \cup (2, \infty)$

2. Partition #1's of $f'$: $f' = \frac{8(x+4)}{(x-2)^4}$
   
   $f' = 0 \Rightarrow x = -4$
   
   $f'$ MNE $\Rightarrow$ bottom $= 0 \Rightarrow x = 2$

3. Critical values of $f$: $x = -4$

4. Sign chart of $f'$:

   $f'$ $\Theta$ $\Theta$ $\Theta$

   $f$ ↓ 4 ↑ 0 ↑

   $
   \downarrow$

   $\Delta u. \text{ on } (-4, 2) \text{ and } (2, \infty)$
   
   $\Delta d. \text{ on } (-\infty, -4)$
   
   Local min at $x = -4$
9. The elasticity of demand for a certain commodity is 
\[ E(p) = \frac{2p^2}{1200 - p^2}, \]
where \( p \) is the unit price in dollars. The current price per item is $30.

a) If this price is increased by $1.20, describe the effect on demand.

\[ \text{Change} \]
\[ \% \text{ chg dem} = E(30) \times \% \text{ chg price} \]

\[ E(30) = 6 \]
\[ \% \text{ chg price} \rightarrow \text{new-old} = \frac{31.20 - 30}{30} = 0.04 \Rightarrow 4\% \]

\[ \Rightarrow \% (4\%) = 24\% \Rightarrow \text{Demand ↓ by 24\%} \]

b) For what values of \( p \) will a percentage change in price cause a smaller percentage change in demand?

\[ E(p) < 1 \Rightarrow \frac{2p^2}{1200 - p^2} < 1 \]
\[ \Rightarrow 2p^2 < 1200 - p^2 \]
\[ \Rightarrow 3p^2 < 1200 \]
\[ \Rightarrow p^2 < 400 \]
\[ \Rightarrow p < 20 \text{ or } p > -20 \]
\[ \Rightarrow (0, 20) \]

i.e. where is demand inelastic?

c) Given the current price is $30, what should the company do to increase revenue?

Since \( E(30) = 6 > 1 \Rightarrow \text{elastic} \)

\[ \Rightarrow \text{ma. rev} \Rightarrow \text{decr. price} \]
10. Sketch an example of a function that has a critical value at $x = 3$ such that $f'(3)$ does not exist and there is a relative maximum at $x = 3$.

\[ f(3) \text{ defined} \hspace{1cm} \begin{align*} f'(3) &\neq 0 \hspace{1cm} \text{at} \ x = 3 \end{align*} \]

11. If a function has a critical value at $x = 5$, does that mean it has a relative max or min at $x = 5$? Explain graphically.

\[ \boxed{\text{No}} \hspace{1cm} \text{Possible to have critical value with no relative (local) extrema.} \]