

MATH 151  
FALL 2007

SAMPLE EXAM III

Part I - Multiple Choice

1. Given  $f(x) = x^3 \ln x$ , find  $f'(e)$

- a)  $e$                       b)  $3 + 3e^2$                       c)  $e^2$   
d)  $3e$                       e)  $4e^2$

2. The population of a bacteria colony triples every 5 hours. If the population follows an exponential growth model, find  $k$ .

- a)  $\frac{\ln 2}{5}$                       b) not enough information  
c)  $\frac{\ln 3}{5}$                       d) 3                      e)  $\ln 5$

3. Evaluate  $\int_{-3}^0 \sqrt{9-x^2} dx$

- a)  $\frac{5\pi}{2}$                       b)  $2\pi$                       c)  $\frac{9\pi}{2}$   
d)  $\frac{9\pi}{4}$                       e) None of the above

4.  $\frac{d}{dx}(\tan^{-1}(x^2)) =$

- a)  $\frac{2x}{1+x^4}$                       b)  $\frac{2}{1+x^2}$                       c)  $-2x \csc(x^2) \cot(x^2)$   
d)  $2x \tan^{-1}(x^2) \sec^{-1}(x^2)$                       e)  $\frac{2x}{1+x^2}$

5. Find the value of  $c$  that satisfies the Mean Value Theorem for  $f(x) = x^2 + 4$  on the interval  $[-1, 2]$ .

a)  $\frac{7}{4}$

b)  $\frac{2}{3}$

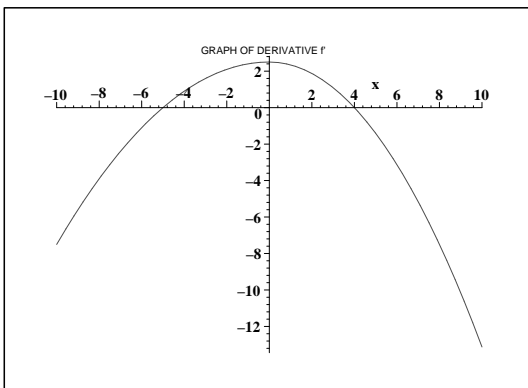
c)  $\frac{1}{3}$

d) 1

e)  $\frac{1}{2}$

6. The graph of the *DERIVATIVE* of a function is shown below. On which intervals is the original function  $f$  concave down?

**CIRCLE ALL CORRECT CHOICES-THERE MAY BE MORE THAN ONE!**



a)  $(-\infty, -5)$

b)  $(-5, 0)$

c)  $(0, 4)$

d)  $(4, \infty)$

e) none of these intervals

7. Circle ALL the critical values of  $f(x) = x(x - 1)^{\frac{1}{3}}$

**NOTE: YOU MAY CIRCLE MORE THAN ONE CHOICE!**

- a) 0                      b)  $-\frac{1}{3}$                       c)  $\frac{1}{4}$
- d)  $\frac{3}{4}$                       e) 1

8. Find the absolute maximum of  $f(x) = \sin x + \cos x$  on the interval  $\left[0, \frac{\pi}{3}\right]$ .

(NOTE:  $\sqrt{2} \approx 1.414$  and  $\sqrt{3} \approx 1.73$ )

- a) 1                      b) 2                      c)  $\sqrt{2}$
- d)  $\frac{\sqrt{3} + 1}{2}$                       e)  $\frac{\pi}{4}$

9. The inflection points of  $f(x) = x^5 + 10x^4$  occur at which of the following?

a)  $x = 6$  only

b)  $x = -6$  only

c)  $x = 0, x = -8$

d)  $x = 0, x = -6$

e)  $x = 0, x = 6$

10. To find the rectangle of perimeter 100 cm with the largest area, you would maximize which function?

a)  $f(x) = x(50 - x)$

b)  $f(x) = 2x + 2(100 - x)$

c)  $f(x) = \frac{100}{x^2}$

d)  $f(x) = 50 - 2x$

e)  $f(x) = 2x + \frac{200}{x}$

11. Which is an antiderivative of  $f(x) = 2\sqrt{x} + \frac{1}{x^2}$ ?

a)  $\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{x} + C$

b)  $\frac{4}{3}x^{\frac{3}{2}} - \ln(x^2) + C$

c)  $\frac{1}{\sqrt{x}} - \frac{2}{x^3} + C$

d)  $\frac{1}{\sqrt{x}} + \ln(x^2) + C$

e)  $\frac{1}{\sqrt{x}} - \frac{1}{x} + C$

12. Write  $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64}$  in summation notation.

a.)  $\sum_{n=1}^6 \frac{(-1)^{n+1}}{2^n}$

b.)  $\sum_{n=1}^6 \frac{(-1)^n}{2^n}$

c.)  $\sum_{n=1}^6 \frac{(-1)^{n+1}}{2n}$

d.)  $\sum_{n=1}^6 \frac{(-1)^n}{2n}$

e.)  $\sum_{n=1}^6 \frac{-1}{2^n}$

13. Find  $\int_0^5 |x - 2| dx$

a.)  $\frac{9}{2}$

b.)  $\frac{7}{2}$

c.)  $\frac{15}{2}$

d.)  $\frac{11}{2}$

e.)  $\frac{13}{2}$

14. Use the midpoint rule with  $n = 4$  to approximate  $\int_1^3 \ln x dx$

a.)  $\ln \frac{3465}{256}$

b.)  $\frac{1}{2} \ln \frac{3465}{256}$

c.)  $\frac{1}{2} \ln \frac{32}{16}$

d.)  $\ln \frac{32}{16}$

e.) None of the above.

15. Find  $\lim_{x \rightarrow 0} \frac{\arctan x - x}{x^3}$

a.)  $-\frac{1}{3}$

b.)  $-\frac{1}{2}$

c.) 0

d.)  $\infty$

e.) -6

16. Find an upper and lower bound on  $\int_0^2 \sqrt[3]{x^2 + 1} dx$ .

a.)  $2 \leq \int_0^2 \sqrt[3]{x^2 + 1} dx \leq 2\sqrt[3]{5}$

b.)  $1 \leq \int_0^2 \sqrt[3]{x^2 + 1} dx \leq \frac{1}{3}\sqrt[3]{5}$

c.)  $2 \leq \int_0^2 \sqrt[3]{x^2 + 1} dx \leq \sqrt{5}$

d.)  $1 \leq \int_0^2 \sqrt[3]{x^2 + 1} dx \leq \frac{1}{2}\sqrt{5}$

e.) Cannot be determined.

## Part II - Work Out Problems

17. Find the derivative of  $f(x) = x^{\sec x} + 2^{\arcsin x}$

18. Find  $\lim_{x \rightarrow 0} (1 - x)^{\frac{5}{x}}$

19. On what intervals is  $f(x) = x + 2 \cos x$ ,  $0 \leq x \leq 2\pi$ , increasing?

20. Find the intervals of concavity for  $f(x) = xe^{4x}$

21. Find the dimensions of the largest rectangle that can be inscribed in a circle of radius 2.

22. The acceleration of a particle is given by  $\mathbf{a}(t) = (1 + e^t)\mathbf{i} + (\cos t)\mathbf{j}$ . If the initial velocity is  $\mathbf{i}$  and the initial position is  $\mathbf{j}$ , find the position of the particle at any time  $t$ .

23. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{1}{x^4} - \frac{1}{x^2} \right)$

24. By introducing a deadly chemical, the population of a particular bacteria culture is decreasing over time. Suppose it has been determined that the rate of change of the population at time  $t$  minutes is  $\frac{1}{3}$  of the population. If the initial size of the population is 1000 bacteria, how many bacteria are present after 3 minutes?

25. Find the following:

a.)  $\arccos\left(\frac{1}{2}\right) = \underline{\hspace{2cm}}$

b.)  $\sin\left(\arccos\left(-\frac{4}{5}\right)\right) = \underline{\hspace{2cm}}$

c.)  $\arcsin\left(\sin\left(\frac{5\pi}{6}\right)\right) = \underline{\hspace{2cm}}$

d.) The domain of  $\arccos(4x - 5) = \underline{\hspace{4cm}}$

e.) The domain of  $\arctan(\ln x) = \underline{\hspace{4cm}}$