Fall 2007 Math 151

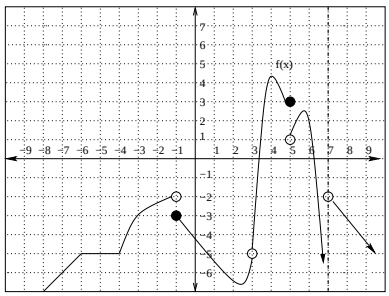
Week in Review 3

courtesy: Amy Austin

(covering sections 2.5 - 2.7)

Section 2.5

1. Referring to the graph, explain why the function $f(x)$ is or is not continuous (you decide which) at $x = -1$, $x = 3$, $x = 5$, $x = -4$ and $x = 7$.



2. Where is the function

$$f(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ 4x & -1 \le x < 1 \\ 3 & x = 1 \\ 5 - x & \text{if } x > 1 \end{cases}$$

not continuous? Fully support your answer.

3. If $f(x) = \dfrac{x+2}{x^2+5x+6}$, find all values of $x = a$ where the function is discontinuous. For each discontinuity, find the limit as x approaches a, if the limit exists. If the limit does not exist, support your answer by evaluating left and right hand limits.

4. Suppose it is known that $f(x)$ is a continuous function defined on the interval $[1, 5]$. Suppose further it is given that $f(1) = -3$ and $f(5) = 6$. Give a graphical arguement that there is at least one solution to the equation $f(x) = 1$.

5. If $g(x) = x^5 - 2x^3 + x^2 + 2$, use the Intermediate Value Theorem to find an interval which contains a solution to the equation $g(x) = -1$.

6. Use the Intermediate Value Theorem to find two consecutive integers a and $a + 1$ such that the interval $[a, a + 1]$ contains a solution to the equation $x^3 + 2x + 1{=}0$.

7. Find the values of c and d that will make

$$f(x) = \begin{cases} 2x & \text{if } x < 1 \\ cx^2 + d & \text{if } 1 \le x \le 2 \\ 4x & \text{if } x > 2 \end{cases}$$

continuous on all real numbers.

Section 2.6

8. Compute the following limits:

a.) $\lim\limits_{x\to\infty} \dfrac{4x^3 - 6x^4}{2x^3 - 9x + 1}$

b.) $\lim\limits_{t\to-\infty} \dfrac{t^9 - 4t^{10}}{t^{12} + 2t^2 + 1}$

c.) $\lim\limits_{x\to\infty} \dfrac{4x - 6x^3}{-2x^3 - 9x + 1}$

d.) $\lim\limits_{x\to\infty} \dfrac{\sqrt{2 + x^2}}{4 - 7x}$

e.) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{5x^2 + 1}}{x - 3}$

f.) $\lim\limits_{x\to\infty} (\sqrt{x^2 + 5x + 1} - x)$

g.) $\lim\limits_{x\to-\infty} (x + \sqrt{x^2 + x + 2})$

9. Find all horizontal and vertical asymptotes of $f(x) = \dfrac{x^3}{x^3 - x}$

Section 2.7

10. Using the limit definition for slope, find the equation of the tangent line to the graph of $f(x)$ at the indicated value:

a.) $f(x) = \sqrt{x - 2}$ at the point $(6, 2)$

b.) $f(x) = \dfrac{1}{1 - x}$ at $x = 0$

11. The displacement (in meters) of a particle moving along a straight path is given by $s(t) = t^2 - 8t + 18$, where t is measured in seconds. Compute:

a.) The average velocity of the particle over the time interval $[1, 2]$.

b.) The instantaneous velocity at time $t = 1$.

12. Given $\mathbf{r}(\mathbf{t}) = \langle 3t - 1, t^2 + 4 \rangle$:

a.) Find the tangent vector to the curve $\mathbf{r}(\mathbf{t})$ at the point $(2, 5)$.

b.) Find parametric equations for the tangent line to the curve at the point $(2, 5)$.

c.) Eliminate the parameter to find a cartesian equation of the tangent line.