

**Fall 2005 Math 151**  
**Exam 2 Review Exercises**

**Night Before Drill**  
*courtesy: Amy Austin*

Review Exercises: Sections 3.4 - 4.3

**Section 3.4**

1. Find the limit:
  - a.)  $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x}$
  - b.)  $\lim_{x \rightarrow 0} \frac{\sin(5x)}{\tan(3x)}$
  - c.)  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x}{3x}$
2. Find the derivative of  $f(x) = \sin x \cos x$
3. Find  $f'(\frac{\pi}{4})$  for  $f(x) = \frac{\sin x}{x}$
4. Find the tangent line to the graph of  $f(x) = \sec x$  at  $x = \frac{\pi}{3}$

**Section 3.5**

5. Find  $f'(x)$  for  $f(x) = \sin^3 4x$
6. Find the tangent line to the graph of  $f(x) = \frac{8}{\sqrt{4+3x}}$  at  $x = 0$ .
7. Find  $f'(0)$  for  $f(x) = (x+1)^2(2-3x)^3$
8. Suppose that  $w = u \circ v$ ,  $u(0) = 1$ ,  $v(0) = 2$ ,  $u'(0) = 3$ ,  $u'(2) = 4$ ,  $v'(0) = 5$  and  $v'(2) = 6$ . Find  $w'(0)$ .
9. If  $G(x) = \cos(f(x)) + f(\cos x)$ , find  $G'(x)$ .

**Section 3.6**

10. Find  $\frac{dy}{dx}$  for  $\cos(x-2y) = x^3y^3 + 1$
11. Find the tangent line to the graph of  $y^2 = x^3(2-x)$  at the point  $(1, 1)$ .
12. If  $6g(x) = xf(x^2) + (g(x))^2$ , find an expression for  $g'(x)$ .

**Section 3.7**

13. Find a unit tangent vector to the curve  $\mathbf{r}(t) = \langle 2t, 3t^3 \rangle$  at  $t = -1$ .
14. A cannonball fired from a cannon has a position function given by  $\mathbf{r}(t) = \langle 5t, 36t - 2t^2 \rangle$ .
  - a.) Compute the velocity and the speed of the cannonball at time  $t = 4$ .
  - b.) With what speed does the cannonball hit the ground?
  - c.) How far does the cannonball travel before hitting the ground?
15. Given  $\mathbf{r}(t) = \langle 2 \cos t, 3 \sin t \rangle$ :
  - a.) Sketch the curve by eliminating the parameter.
  - b.) Find the position, tangent (velocity), and acceleration vector to the curve at  $t = \frac{\pi}{3}$ .
  - c.) Find parametric equations for the tangent line to the curve  $\mathbf{r}(t)$  at  $t = \frac{\pi}{3}$ .

**Section 3.8**

16. Find  $y''$  for  $y = \sin^2 x$ .
17. If  $\mathbf{r}(t) = \langle t^3, t^2 \rangle$  represents the position of a particle at time  $t$ , find the angle between the velocity and the acceleration vector at time  $t = 1$ .

**Section 3.9**

18. Given  $x = \cos t$  and  $y = \sin t \cos t$ , find the equation of the tangent line at  $t = \frac{3\pi}{2}$ .
19. Let  $x = t^4 - 4t^3$  and  $y = 3t^2 - 6t$ .
  - a.) Find the equation of the tangent line at the point  $(5, 9)$ .
  - b.) Find all point(s) on the curve where the tangent line is vertical or horizontal.
20. At what points on the curve  $x = t^3 + 4t$ ,  $y = 6t^2$  is the tangent line parallel to the line  $x = -7t$ ,  $y = 12t - 5$ ?

### Section 3.10

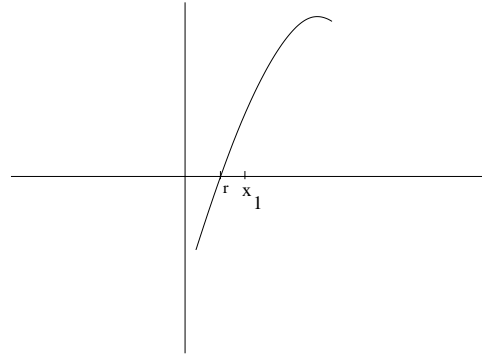
21. A policeman with radar is 0.3 miles off a straight road. He observes a car moving along the road. At the instant when the distance between the car and the policeman is 0.5 miles, that distance is increasing at a rate of 50 mph. How fast is the car moving at that instant?
22. A cylindrical can is undergoing a transformation in which the radius and height are varying continuously with time  $t$ . The radius is increasing at a rate of 4 inches per minute while the height is decreasing at a rate of 10 inches per minute. Is the volume increasing or decreasing, and at what rate, when the radius is 3 inches and the height is 5 inches?
23. A television camera is positioned 4000 feet from the base of a rocket launching pad. A rocket rises vertically and its speed is 600 feet per second when it has risen 3000 feet. If the television camera is always kept focused on the rocket, how fast is the camera's angle of elevation changing at that moment?
24. Suppose we have a circular cone with height 1 foot and radius 3 feet, vertex at the bottom. Suppose further that a solution is being poured into the cone at a rate of 0.5 cubic feet per minute. How fast is the height of the solution increasing when the volume of the solution is  $\frac{3\pi}{8}$  cubic feet?

### Section 3.11

25. Given  $y = 4 - x^2$ 
  - a.) Find  $\Delta y$  if  $x$  changes from  $x = 1$  to  $x = 1.5$
  - b.) Find  $dy$  for  $x = 1$  and  $dx = 0.5$ .
26. Use differentials to approximate  $\sqrt{26}$ .
27. The radius of a circular disk is measured to be 30 cm with a maximum error in measurement of .2 cm. Use differentials to approximate the maximum error in the calculated area of the disk.
28. Find the linearization for  $f(x) = \sqrt{x+3}$  at  $x = 1$  and use it to approximate  $\sqrt{3.98}$ .
29. Find the quadratic approximation to  $f(x) = \frac{1}{x+2}$  at  $x = 1$ .

### Section 3.12

30. Use Newton's Method with initial guess  $x_1 = 2$  to find  $x_2$ , the second approximation, to a root of the equation  $x^5 - 34 = 0$ .
31. Suppose we wish to use Newton's method to approximate the root  $r$ . We begin with an initial guess  $x_1$ . Give the location of  $x_2$ .



### Section 4.1

32. Compute  $\lim_{x \rightarrow \infty} \frac{2^x - 3^{-x}}{4^x + 5^{-x}}$
33. Find  $\lim_{x \rightarrow 2^+} 5^{\frac{x-1}{2-x}}$
34. Find  $f'(1)$  for  $f(x) = e^{e^{2x}}$
35. For what values of  $r$  does the function  $y = e^{rx}$  satisfy the equation  $y'' + 2y' - 3y = 0$ ?
36. Find  $g'(t)$  for  $g(t) = \sin(e^{\sqrt{t}} + 1)$
37. Find  $f^{(21)}(x)$  for  $f(x) = e^{\frac{1}{3}x}$

### Section 4.2

38. Given  $f(x) = 2x^3 + 6x + 2$  and  $g(x)$  is the inverse of  $f(x)$ :
  - a.) Compute  $g(10)$
  - b.) Compute  $g'(10)$
39. Prove  $f(x) = \sqrt[3]{2x+5}$  is one-to-one and find the inverse.
40. Find the inverse of  $f(x) = \frac{x-2}{x+2}$ .
41. Suppose  $g$  is the inverse of a function  $f$  and  $f(4) = 5$ ,  $f'(4) = -3$ , find  $g'(5)$ .

### **Section 4.3**

42. Evaluate  $\log_3 108 - \log_3 4$
43. Express  $\log_8 x - \log_8 \sqrt{9x + 2} + \log_8(x + 1)$  as a single logarithm.
44. Solve for  $x$ :  $\log(x + 3) + \log(x) = 1$
45. Solve for  $x$ :  $y = \ln(7x - 9)$
46. Solve for  $x$ :  $\ln x - \ln(x + 1) = \ln 2 + \ln 3$
47. Find the inverse of  $f(x) = e^{6x-3}$
48. Find  $\lim_{x \rightarrow \infty} [\log(2x - 1) - \log(3x + 6)]$
49. Find the value of  $\ln \sqrt{e^3}$
50. What is the domain of  $f(x) = \ln(4 - x^2)$ ?