Fall 2007 Math 151
WIR7 [reformatted and augmented]
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1. Given a graph of f' (or f''), talk about f.
   See appropriate Maplets!

2. Sketch the graph of a function f with these properties.

   - Domain: All real numbers
   - $f(-1) = -2,\ f(0) = 0,\ f(2) = 3$
   - $f'(x) < 0$ for $(-\infty, -1) \cup (2, \infty)$
   - $f'(x) > 0$ for $(1, 2)$
   - $f''(x) > 0$ for $x < 0$ and $f''(x) < 0$ for $x > 0$

3. Identify all critical values of the following functions.

   (a) $f(x) = xe^{2x}$
   (b) $f(x) = \left|x^2 - 2x\right|$
   (c) $f(x) = \left(x^2 - x\right)^{1/3}$

4. Find absolute and local extrema via graphing.

   (a) $f(x) = 1 - x^2, -1 \le x \le 2$
   (b) $f(x) = \begin{cases} x^2 & \text{if } -1 \le x < 0 \\ 2 - x^2 & \text{if } 0 \le x \le 1 \end{cases}$

5. In each case, find the absolute extrema.

   (a) $f(x) = x^3 - 12x + 1$ on $[-1, 5]$
   (b) $f(x) = x \ln x$ on $[1, 3]$

6. In each case, sketch the graph of a function f with the specified properties.

   (a) $x = 2$ is a critical number of f, but f has no local extrema
   (b) f is continuous with a local maximum at $x = 2$, but f is not differentiable at $x = 2$
   (c) f is defined on $[2, 5]$, but has no absolute maximum

7. State the Mean Value Theorem (MVT), then find all numbers c that satisfy the conclusion of the MVT for $f(x) = x^2$ on $[-1, 2]$.

8. Find the intervals where the given function is increasing or decreasing and indentify all local extrema.

   (a) $f(x) = 3x^4 + 4x^3 - 12x^2 + 8$
   (b) $y = \tan^{-1}\left(x^2\right)$
   (c) $f(x) = \dfrac{x}{(x-1)^2}, x \ne 1$
   (d) $f(x) = x \sin x + \cos x$ on $[0, 2\pi]$

9. Determine the intervals where $f(x) = x^5 + 5x^4$ is concave up or concave down and identify all inflection points.

10. Given the following information, identify any local extrema.

x	$f(x)$	$f'(x)$	$f''(x)$
-3	4	0	7
2	-5	0	-6

11. A poster is to have an area of 180 in.^{2} with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions give the largest printed area?

12. A company wants to manufacture a rectangular box with a volume of 36 ft^3. The box has no top and its length is twice its width. Find the dimensions of the box that mimimize the amount of material used.

13. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of length 3 cm and 4 cm if two sides of the rectangle lie along the legs.

14. In each case find the most general antiderivative.

   (a) $f(x) = 5x^{1/4} - 7x^{3/4}$
   (b) $g(\theta) = \cos\theta - 5\sin\theta$
   (c) $f(\theta) = 6\theta^2 - 7\sec^2\theta$
   (d) $f(t) = 2\sec t \tan t + \frac{1}{2}t^{-1/2}$
   (e) $g(x) = \left(\dfrac{1-x}{x}\right)^2$
   (f) $f(x) = \dfrac{(1+e^x)^2}{e^x}$

15. Find f given $f''(t) = 3t^{-1/2}$, $f(4) = 20$, and $f'(4) = 7$.

16. Given acceleration $\mathbf{a}(t) = \left[e^t, 4t + \cos t\right]$, initial velocity $\mathbf{v}(0) = [2, -1]$, and initial position $\mathbf{r}(0) = [0, 5]$, find the position at time $t = 2$.