

Problems for the LIFE review; Wednesday 12/5 10:15–12:15 HELD 200

1. If P is the point (2,6) and Q is the point (-1,2), give a unit vector orthogonal to the vector from Q to P. Also, find a vector equation of the line passing through P and Q.

2. Let  $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j}$  and  $\mathbf{b} = \mathbf{i} - 4\mathbf{j}$ . What vector added to  $2\mathbf{a}$  yields  $\mathbf{b}$ ?

3. Find the value(s) of  $c$  that makes

$$f(x) = \begin{cases} cx^2 - 1 & \text{for } x < 3 \\ 5 - cx & \text{for } x \geq 3 \end{cases}$$

continuous for all real  $x$ .

4.  $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 4}}{4x + 1} =$

5. For which value(s) of  $r$  is  $y = e^{rt}$  a solution to the differential equation  $2y'' - y' - 3y = 0$ ?

6. Given  $y = (\sin x)^{\frac{1}{x}}$ , what is  $y'$ ?

7.  $\tan(\sin^{-1} x) =$

8. A closed can is to be constructed in the shape of a cylinder. If the surface area of the can (including the top and bottom of the can) is fixed at 60 square feet, find the radius and height of the can with maximum volume.

9. Given  $f'(x) = x - 3 \sin x$  and  $f(0) = -2$ , what is  $f(\pi)$ ?

10.  $d/dx \left( \int_0^{\sin x} \sqrt{4 + 5t^4} dt \right) =$

11. Approximate  $\int_{-2}^4 (16 - x^2) dx$  using a partition of  $P = \{-2, 0, 4\}$  and taking  $x_i^* = \text{midpoint}$ .

12. Find the equation of the line tangent to the curve  $\mathbf{r}(t) = \left\langle te^{-t}, \frac{1}{\sqrt{t}} \right\rangle$  at the point  $\left(\frac{1}{e}, 1\right)$

13. Find  $y'$  for each of the following:

a)  $y = \frac{x}{\cos^2(4x)}$

b)  $\arcsin y = \sqrt{x}$

14. Find the limit:

a)  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

b)  $\lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{x^2}$

c.)  $\lim_{t \rightarrow 0} \frac{\sin(5t)}{t}$

15. Sand is being dumped at a rate of  $100 \text{ cm}^3$  per second onto a pile in the shape of a cone whose diameter is one-half its height. How fast is the height of the pile increasing when the height is 75 cm?

16. Compute each of the following:

a)  $\int \left( \frac{3}{\sqrt{1-x^2}} + \frac{x}{3} + \frac{3}{x} \right) dx$

b)  $\int \frac{x}{x^2+1} dx$

c.)  $\int_0^1 \frac{3x - 1}{(3x^2 - 2x + 1)^4} dx$

d.)  $\int \frac{x}{\sqrt{x+4}} dx$

17. A triangle has vertices  $A(-3, -2)$ ,  $B(1, 2)$ , and  $C(0, 4)$ . Find the angle of the triangle at vertex  $A$ .

18. Find the distance from the point  $(1, 3)$  to the line  $y = x + 5$ .

19. Find the linear approximation of  $f(x) = \sqrt[3]{x}$  at  $x = \frac{27}{8}$  and use it to approximate  $\sqrt[3]{3}$

20. Find  $f'(x)$  for  $f(x) = |9 - x^2|$ .