Section 4.4

1. Differentiate each function:
   a.) \( f(t) = \cos^2 t \ln t \)

   b.) \( f(x) = \ln(\sin 2x) \)

   c.) \( h(x) = \ln(\ln 3x) \)
d.) \[ y = \ln \left( \frac{x^2 + 1}{3x - 5} \right) \]

e.) \[ f(x) = \log_5(e^{10x}) \]

f.) \[ f(x) = 3^{\tan(7x)} \]
g.) \( y = x^{\sin x} \)

2. Find the equation of the tangent line to the graph of \( f(x) = x \ln x \) at \( x = e^2 \).
3. What is the slope of the parametric curve
\[ x = t \ln t, \quad y = 2^{3t} \] at the point \((0, 8)\)?
Section 4.5

4. A bacteria culture starts with 400 bacteria and the population triples every 20 minutes.
   a.) Find an expression for the number of bacteria after $t$ hours.
   b.) Find the number of bacteria after 2 days.
   c.) When will the population reach 20,000?
5. A curve that passes through the point \((0,25)\) has the property that the slope at every point \((x,y)\) is eight times the \(y\) coordinate. Find the equation of the curve.
6. A tank contains 200 liters of brine with a concentration of 0.1 kg of salt per liter. Pure water enters the tank at a rate of 5 liters per minute. The solution is kept mixed and exits the tank at the same rate. How many kg of salt will remain after half an hour?
7. A pie is taken from an oven, where the temperature is 450°, to a 75° room. After 15 minutes, the temperature of the pie reads 350°. What will the temperature of the pie be after 27 minutes?
Section 4.6

8. Compute the following without the aid of a calculator.

a.) $\arcsin \frac{\sqrt{3}}{2}$  

b.) $\arccos \left( -\frac{1}{\sqrt{2}} \right)$

c.) $\sin^{-1} \left( -\frac{\sqrt{2}}{2} \right)$  

d.) $\arctan \frac{1}{\sqrt{3}}$

e.) $\cot \arccos \left( -\frac{3}{5} \right)$  

f.) $\sin(\arcsin 2)$
g.) \( \arccos(\cos(\frac{2\pi}{3})) \)  

h.) \( \arctan(\tan(\frac{5\pi}{4})) \)

i.) \( \cos(\arccos 0.4) \)  
j.) \( \arcsin(\sin(\frac{11\pi}{6})) \)

k.) \( \arccos(\cos \frac{5\pi}{4}) \)  
l.) \( \sin(2 \arccos(\frac{1}{3})) \)
9. Find the derivative of \( y = \arctan(1 - x) \)

10. Find the equation of the tangent line to the graph of \( y = \arcsin \frac{x}{2} \) at \( x = -1 \).
11. What is the domain of $f(x) = \arcsin(2x - 1)$?
Of $\arctan(2x - 1)$?

12. Find $\lim_{x \to \infty} \arctan x$.

13. $\cos(\arctan x)$ is equivalent to what?
Section 4.8

14. Find the following limits.

a.) \( \lim_{x \to \infty} \frac{(\ln x)^2}{x - 1} \)

b.) \( \lim_{x \to 0} \frac{\sin x - x}{x^3} \)
c.) \( \lim_{x \to 0^+} x^2 \ln x \)

d.) \( \lim_{x \to \infty} (e^x + x) \frac{1}{x} \)
e.) \( \lim_{x \to 0} (1 + \sin 4x)^{\cot x} \)

f.) \( \lim_{x \to 0} (\sin x)^{\tan x} \)
g.) \( \lim_{x \to 1} \left( \frac{1}{\ln x} - \frac{1}{x - 1} \right) \)