Homework 10

Math 147, Fall 2023

This homework is due on Friday, October 27 (at the start of recitation). *Turn in (via Gradescope) your answers to questions 1–3.*

0. Read Section 5.1. After reading these sections, you should be able to answer the following questions (which are *not* to be turned in).
   - Complete the following sentence: *$f(x)$ has a local maximum at $x = a$ if and only if $-f(x)$ has a __________ at $x = a$.*
   - The Mean-Value Theorem guarantees (under certain hypotheses) the existence of a number c with $a < c < b$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. Does it tell you where in the interval (a, b) the number c is, or how many such c exist? Could there be two? Could there be infinitely many? (Consider a straight line.)
   - What does Rolle's Theorem say? How is it related to the Mean-Value Theorem?

1. For each of the following, give an example of such a function (a sketch of the graph with extrema labeled is fine) OR explain briefly why no such function exists.
   (a) a function with 1 local maximum, 1 local minimum, and no global extrema
   (b) a function with 2 local minima, 1 global minimum, and no local maxima
   (c) a function with no local extrema
   (d) a function with infinitely many global extrema

2. Find all **critical numbers**[1] of the following functions:
   (a) $f(x) = x^3 + 6x^2 + 3x - 1$
   (b) $g(x) = x + \sin x$
   (c) $h(x) = \frac{1}{(1-x)^2}$

3. Section 5.1 #6, 10, 12, 16, 18

4. (These problems are *not* to be turned in!) Section 5.1 # 1, 3, 5, 9, 11, 13, 18, 19, 25, 29, 33,

[1] A **critical number** is some number c at which $f'(c) = 0$ or $f'(c)$ does not exist.