Homework 4

Math 147, Fall 2023

This homework is due on Friday, September 15 (at the start of recitation). *Turn in (via Gradescope) your answers to questions 1–7.*

Hint: If you do not have a graphing calculator, use this one online: `www.desmos.com/calculator`

0. Read Sections 3.2, 3.3, 3.4

1. For each of the following functions $h(x)$, determine the domain and where (at which points) the function is continuous. Additionally, find functions $f(x)$ and $g(x)$ such that $h(x) = f \circ g(x)$. *Recall that* $f \circ g(x) := f(g(x))$.

   (a) $h(x) = \cos\left(\frac{x^2-3}{1-x}\right)$

   (b) $h(x) = \log_3(1-x)$

2. Section 1.3 # 18

3. Are there real numbers a and b for which the following function $f(x)$ is continuous? If so, then determine a and b, and sketch a graph of $f(x)$. If not, then explain why not.

$$f(x) = \begin{cases} -1 & \text{if } x \leq -1 \\ ax+b & \text{if } -1 < x < 1 \\ 5 & \text{if } x \geq 1 \end{cases}$$

4. Evaluate the following limits. Show your work.

   (a) $\lim\limits_{x\to\infty} -3x^5 + 6x$

   (b) $\lim\limits_{x\to-\infty} xe^{-x}$

   (c) $\lim\limits_{x\to\infty} \frac{3x^3+2x^5-1}{-x^2+5}$

   (d) $\lim\limits_{x\to\infty} \frac{x^5+8}{-2x^2+6x^3}$

   (e) $\lim\limits_{x\to0^-} \frac{\cos x}{x}$

   (f) $\lim\limits_{x\to0} \frac{\cos x}{x}$

   (g) $\lim\limits_{x\to0} 2x^3 \cos x$

   (h) $\lim\limits_{x\to\infty} \frac{\sin x}{x^3+6}$

5. Section 3.2 # 8, 28, 49
6. Section 3.3 # 20, 28
7. Section 3.4 # 4, 10, 12, 16
8. (These problems are *not* to be turned in!)
   (a) Section 1.3 # 16
   (b) Section 3.2 # 5, 7, 11, 15, 20, 23, 41, 45
   (c) Section 3.3 # 1, 3, 5, 8, 13, 21, 25, 29
   (d) Section 3.4 # 2, 5, 11, 13, 15, 17
9. (These problems are *not* to be turned in!) For each function below, determine the value(s) (if any) of a that make $f(x)$ continuous.
   (a)
   $$f(x) = \begin{cases} a & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases}$$
   (b)
   $$f(x) = \begin{cases} e^x & \text{if } x < 0 \\ ax & \text{if } x \geq 0 \end{cases}$$