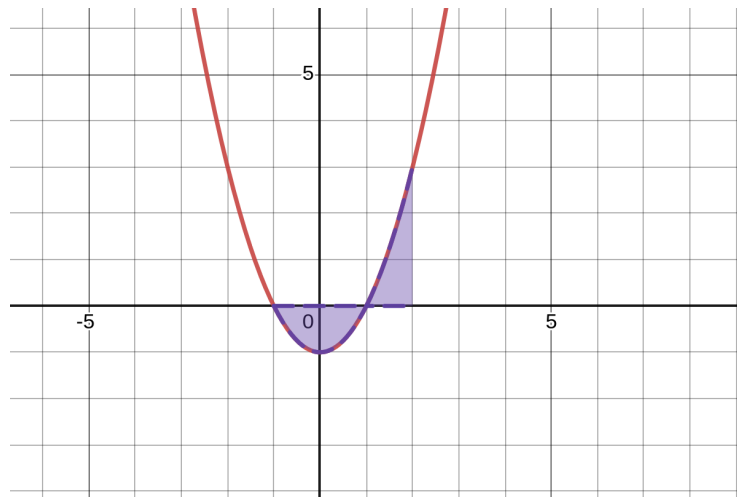


Homework 15

Math 171H (section 201), Fall 2023

This homework is due on **Thursday, November 30** at the start of class. (Turn in answers to questions 1–7.)

0. Read Sections 5.4–5.5
 - (a) Explain the difference between definite and indefinite integrals.
 - (b) How is the definite integral $\int_a^b f(x)dx$ related to the indefinite integral $\int f(x)dx$?
1.
 - (a) Sketch the region under the curve $y = \sqrt{x}$, for $0 \leq x \leq 16$. Compute the area.
 - (b) Sketch the region bounded by $y = x^2$ and $y = 18 - x^2$. Compute the area.
 - (c) Compute $\int_0^1 (1 + s)^3 ds$.
2. Compute the definite integral $\int_1^3 (3 - x)dx$ by using the Fundamental Theorem of Calculus. (Does your answer match your answer from HW 14?)
3. Compute the area of the shaded region shown below (use your best guess for what the function is).



4. Consider the function $f(x) = \int_0^x 1 + \sin(\sin t)dt$.
 - (a) Show that $f(x)$ is an increasing function. Conclude that $f^{-1}(x)$ exists.
 - (b) Compute $\frac{df^{-1}}{dx}$ at 0.
5. Is there a continuous function f for which

$$\int_1^x f(t)dt = (f(x))^2 + 8.1 ?$$

If so, find all such functions. If not, explain why not.

6. Find a function g for which

$$\int_2^{x^2} tg(t)dt = x + x^2,$$

or explain why no such function exists.

7. Compute the following definite integrals.

(a) $\int_0^2 \cos(\pi\theta/2)d\theta$

(b) $\int_0^2 (4t - 2)^{21} dt$

(c) $\int_0^1 \sqrt{t/2 - 1} dt$

(d) $\int_0^2 \frac{dx}{1-2x}$

(e) $\int_0^{\pi/4} \frac{\sin t}{\cos^2 t} dt$

(f) $\int_0^1 xe^{-x^2} dx$

(g) $\int_{-\pi/4}^{\pi/4} -x^2 \sin x dx$