Homework 5

Math 171H (section 201), Fall 2023

This homework is due on **Tuesday, September 19** at the start of class. (Turn in answers to questions 1–11.)

0. Read Sections 2.6–2.8, including the topic of *horizontal asymptotes* (page 128).

1. Give an example of a function with a horizontal asymptote $y = 10$ and a vertical asymptote at $x = -1$. Briefly justify your answer.

2. State a definition for the following.

   (a)
   $$\lim_{x \to -\infty} f(x) = L$$

   (b)
   $$\lim_{x \to \infty} f(x) = -\infty$$

3. Let n be a positive integer. Compute the following limit (and verify your answer using the definition):
   $$\lim_{x \to \infty} x^n$$
   Hint: Consider two cases, based on whether n is even or odd.

4. (Uniqueness of limits) Prove that if $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} f(x) = M$, then $L = M$.

5. Read the Squeeze Theorem (page 101), and then use it to compute the following limit:
   $$\lim_{x \to \infty} e^{-x} \cos(x)$$

6. For each of the following functions $h(x)$, determine the domain and where (at which points) the function is continuous. Additionally, find functions $f(x)$ and $g(x)$ such that $h(x) = f \circ g(x)$. (*Recall that* $f \circ g(x) := f(g(x))$.)

   (a) $h(x) = \cos\left(\frac{x^2-3}{1-x}\right)$

   (b) $h(x) = \log_3(1 - x)$

7. Determine the domain and where the function $f(x) = \ln(\ln x)$ is continuous. Compute $\lim_{x \to \infty} f(x)$, and prove your answer using the definition.

8. If the tangent line to $y = f(x)$ at $x = 4$ passes through the points $(3, 0)$ and $(5, 4)$, then what are $f(4)$ and $f'(4)$? (Show your work.)

9. The following is the derivative of a function $f(x)$ at some number $x = a$:

$$\lim_{h \to 0} \frac{e^{3(2+h)} - e^{3 \cdot 2}}{h} .$$

Determine the function $f(x)$ and the number a.

10. Use the limit definition to compute the derivative of the following functions:

    (a) $f(x) = c$ (a constant function)

    (b) $f(x) = x$

11. Use the limit definition to prove the following: *If $f(x)$ and $g(x)$ are differentiable at $x = a$, then the function $f(x) + g(x)$ is too.*