

Homework 6

Math 171H (section 201), Fall 2023

This homework is due on **Wednesday (not Tuesday), September 27** at the start of recitation. (Turn in answers to questions 1–7.)

0. Read Sections 3.1–3.2

1. (a) Use the limit definition to compute the derivative of $\frac{1}{g(x)}$.
(b) Use (a) to compute the derivative of x^n , for negative integers n .
(c) Prove the quotient rule, using (a) and the product rule.
2. Let r be a positive integer, and let c be a positive real number. Consider the polynomial $f(x) = cx^r - 6x^{r-1} - 6x^{r-2} - \dots - 6x - 6$.
(a) Evaluate $\lim_{x \rightarrow \infty} f(x)$.
(b) Use the Intermediate-Value Theorem to explain why $f(x)$ has a positive root.
3. If $f(x) + g(x)$ is differentiable at $x = a$, does it mean that both $f(x)$ and $g(x)$ are too? Justify your answer.
4. Consider the following function:

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{else} \end{cases}$$

- (a) Is $f(x)$ differentiable at 0? Prove your answer. (*Hint*: Squeeze Theorem.)
- (b) Is $f(x)$ differentiable at -1 ? Prove your answer.
5. Evaluate the derivatives of the following functions:
 - (a) $3.1x^5 - 16x + 2$
 - (b) $\frac{2x-1}{1-x}$
6. Give two examples of functions $f(x)$ for which $f'(x) = 3x^2 - 0.5x$.
7. Determine the value(s) of m and b that make the following function differentiable:

$$f(x) = \begin{cases} x^3 - x & \text{if } x \geq 0 \\ mx + b & \text{if } x < 0 \end{cases}$$

If your function twice-differentiable (that is, does the second derivative exist)? If so, compute it. (Show your work.)