

Homework 3

Math 300, Fall 2022

This homework is due on Friday, September 9.

0. (*This problem is not to be turned in.*) Read Sections 1.1 and 1.2.

1. Read (in your textbook) about *tautologies* and *contradictions*. Determine whether the following statement is a tautology, a contradiction, or neither (prove your answer):

$$\neg((\neg Q \vee (P \Rightarrow Q)) \Rightarrow (\neg P)) .$$

2. Consider the following statement: *If $x \in \mathbb{R}$, then $x \in \mathbb{Z}$.* Is it true? Is the converse true? Is the contrapositive true? (Explain your answers.)

3. Express the following statements using symbols (such as \forall , \exists) and (as appropriate) “such that”:

(a) Every even integer can be expressed as the sum of two odd integers.

(b) The product of any three odd integers is odd.

(c) The square of at least one real number is 0.

(d) There is an integer whose square is negative.

4. Negate your answers to #3.

5. Prove or disprove each statement in #3.

6. A student turns in the following proof: *Assume that p , q , and r are even integers. Then, by definition, there exists an integer k such that $p = 2k$ and $q = 2k$ and $r = 2k$. Therefore, $pqr = (2k)(2k)(2k) = 8k^3 = 2(4k^3)$, by associativity and commutativity of integers. By closure, $4k^3$ is an integer and hence pqr is even (by definition).*

(a) What statement (your best guess) was this student trying to prove?

(b) Critique the proof.

7. Critique the following “proof” that every even integer is also odd: *Assume that x is even. Then, by definition, $x = 2k$ for some integer k . So, $x = 2k = 2(k - 1/2) + 1$. Hence, by definition, x is odd.*

8. Section 1.1 #7 (Here, a “universe” means a set D so that “ $\forall x$ ” means “ $\forall x \in D$ ”)

9. Section 1.2 #1(b), 2(a), 4

Writing Assignment 3

Math 300

This homework is due on Friday, September 9. *Complete this part on a separate piece of paper, not the same paper for Homework 3 (do not staple the two homeworks together).*

1. Read “Writing Mathematics” (pp. 209–210) in your textbook. Which rules (if any) surprised you?
2. Which topic did you pick for your final paper? Write two paragraphs describing why you picked it, what you know (so far) about the topic, and what you expect to learn.
3. How are you feeling (in general OR in this class OR etc.) at this point in the semester? Answer this question in **one** of the following ways: (a) a paragraph, (b) a poem or picture, (c) a video (emailed to the instructor), or (e) one or more slides – e.g. made using Powerpoint, Google slides, or Beamer (emailed to the instructor)