

# Homework 6

Math 300, Fall 2022

This homework is due on Friday, September 30. (Turn in your answers to questions 1–6.)

0. (*This problem is not to be turned in.*) Read Sections 2.3, 2.4, 4.1
  - (a) Section 4.1 #1, 2
  - (b) What is the difference between these:  $\in$  and  $\subseteq$ ?
  - (c) *Prove or disprove:* For real numbers  $x$  and  $y$ , if  $xy \neq 0$ , then  $x \neq 0$ .
1. Prove or disprove the following claims:
  - (a) Every odd integer can be expressed as the product of two odd integers.
  - (b) Every even integer can be expressed as the product of two even integers.
  - (c) Let  $n$  be an integer. If  $2|(n^2 - 5)$ , then  $4|(n^2 - 5)$ .
  - (d) Let  $n$  be an integer. If  $2|(n^2 - 5)$ , then  $8|(n^2 - 5)$ .
2. Is there something wrong with this supposed proof? If so, identify all the errors, and then either prove or disprove the claim. If not, explain why the proof is complete.  
**Claim:** The average of three even numbers is an even number.  
**Proof:** We proceed by contradiction: assume that the average of three even numbers is odd. However, the average of 2, 4, and 6, which is 4, is even. This is a contradiction.
3. True/False (explain your answers briefly)
  - (a) For every set  $A$ , the following holds:  $\emptyset \subseteq A$ .
  - (b) For every set  $A$ , the following holds:  $\emptyset \in A$ .
  - (c) For every set  $A$ , the following holds:  $\{\emptyset\} \subseteq A$ .
  - (d)  $[4, 6] \subseteq (4, 5)$
4. Rewrite the following sets as lists:
  - (a)  $\{n \in \mathbb{Z} \mid 2 < n \leq 5\}$
  - (b)  $\{n \in \mathbb{R} \mid n^2 = 100\}$
  - (c)  $\{n \in \mathbb{Z} \mid n^2 \leq 30\}$
  - (d)  $[3, 10] \cap \mathbb{Z} \cap \{n \in \mathbb{R} \mid n > 5\}$
5. Prove or disprove the following claims:
  - (a) Let  $A$ ,  $B$ , and  $C$  be sets. If  $A \cap B = A \cap C$ , then  $B = C$ .
  - (b) Let  $A$ ,  $B$ , and  $C$  be sets. If  $A \setminus B = C \setminus B$ , then  $A = C$ .
6. Section 4.1 #3, 4