Homework 8

Math 300, Fall 2022

This homework is due on Friday, October 14[1]. (Turn in your answers to questions 1–7.)

0. *(This problem is not to be turned in.)* Read Sections 3.2, 5.1, and 5.3.

   (a) Explain what is wrong with the following: *Consider a function $f : \mathbb{Z} \to 9$.*

   (b) Explain what is wrong with the following: *Consider a function $f : \mathbb{Z} \mapsto \mathbb{R}$.*

   (c) Give an example of a function $f : \mathbb{Z} \to \mathbb{R}$.

   (d) Give an example of a function $f : \mathbb{R} \to \mathbb{Q}$.

   (e) Determine whether each of the following sets is the graph of some function. Prove your answers.

       (i) $\{(x, y) \in \mathbb{R}^2 \mid x = y^2\}$

       (ii) $\{(x, y) \in \mathbb{Z}^2 \mid x = y - 5\}$

   (f) Section 3.2 #2, 3

   (g) Section 5.1 #1

   (h) Section 5.3 #1, 3, 6, 7, 8

1. (No proofs necessary for this problem)

   (a) List *all* functions $f : \mathbb{Z} \to \{8\}$ (functions with domain $\mathbb{Z}$ and codomain $\{8\}$).

   (b) List *all* **one-to-one** (injective) functions $f : \{0, 1\} \to \{2, 3, 4\}$.

   (c) List *all* **onto** (surjective) functions $f : \{0, 1\} \to \{2, 3\}$.

2. Consider the function $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(n) = 2n$ if n is even and $f(n) = n - 3$ if n is odd.

   (a) *Prove or disprove*: f is one-to-one.

   (b) *Prove or disprove*: f is onto.

3. Let $f : A \to C$ and $g : B \to D$ be functions. Consider the following function:

$$h \;:\; A \times B \to C \times D$$
$$(a, b) \mapsto (f(a), g(b)) \,.$$

   (a) *Prove or disprove*: If f and g are one-to-one, then so is h.

   (b) *Prove or disprove*: If f and g are onto, then so is h.

4. Let A be a nonempty set. Assume $b \notin A$. Consider the following function:

$$h \;:\; \mathcal{P}(A) \to \mathcal{P}(A \cup \{b\})$$
$$S \mapsto S \cup \{b\} \,.$$

   (a) *Prove or disprove*: h is one-to-one.

   (b) *Prove or disprove*: h is onto.

   (c) Is h bijective? Explain your answer.

5. Section 3.2 #1

6. Section 5.1 #2

7. Section 5.3 #3(a)

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[1] As a reminder, your 2+ page draft is also due on October 14.