This homework is due on Thursday, November 19. You may cite results from class or previous homeworks/exams.

0. (This problem is not to be turned in.)
   (a) Read Section 19.
   (b) Section 18 #22, 48 (this subring criterion builds on the earlier subgroup criterion), 49
   (c) Section 19 #18, 24, 27

1. True/false (No proofs necessary for this problem.)
   (a) If $R$ and $R'$ are any two rings, then there exists a ring homomorphism $R \to R'$.
   (b) The rings $\mathbb{Z}$ and $\mathbb{R}$ are isomorphic (as rings).
   (c) $\mathbb{Z}_8^*$ (the group of units of $\mathbb{Z}_8$) is isomorphic to the Klein 4-group.
   (d) $\mathbb{Z}_{10}^*$ is isomorphic to the Klein 4-group.
   (e) $\mathbb{Z}_6^*$ is isomorphic to the Klein 4-group.

2. For each function below, determine whether it is a ring homomorphism. Give a proof.
   (a) $\phi: \mathbb{Z} \to \mathbb{Z}_{10}$ given by $n \mapsto 6n$
   (b) $\psi: \mathbb{Z} \to \mathbb{Z}_{10}$ given by $n \mapsto 5n$
   (c) $\gamma: \{\text{functions } \mathbb{R} \to \mathbb{R}\} \to \mathbb{R}$ given by $f \mapsto f(5) - f(6)$.

3. What are the units of $\mathbb{Z} \times \mathbb{Q} \times \mathbb{Z}_4$? What are the zero divisors? (No proof necessary.)

4. In $M_2(\mathbb{Z})$, is $\begin{pmatrix} 1 & 2 \\ -2 & 0 \end{pmatrix}$ a unit? Is $\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$? Is either a zero divisor? (Explain your answers.)

5. (a) Are $\mathbb{Z}$ and $2\mathbb{Z}$ isomorphic rings? Give a proof.
   (b) Are $2\mathbb{Z}$ and $5\mathbb{Z}$ isomorphic rings? Give a proof.

6. Read about the characteristic of a ring (pages 181–182).
   (a) Define the characteristic.
   (b) State an equivalent definition for rings with unity.
   (c) Determine the characteristics of each of the following rings: $\mathbb{Z} \times \mathbb{Q}$, $\mathbb{Z}_5[x] := \{\text{polynomials with coefficients in } \mathbb{Z}_5\}$, $2\mathbb{Z}$, and $M_n(\mathbb{Z})$. (No proof necessary, but show your work.)

7. Section 19 #17, 26, 29; Section 18 #50 (you may use the subring criterion from #48)