This homework is due on Thursday, October 1. You may cite results from class or previous homework, as appropriate.

0. (This problem is not to be turned in.)
(a) Read Sections 8–9.
(b) Section 8 # 10, 19, 36
(c) Give an example of a non-cyclic group for which all of its proper subgroups are cyclic.
(d) Explain in your own words what a finitely generated group (or subgroup) is.
(e) Is \( \mathbb{R} \) a finitely generated group?

1. Prove that if a group \( G \) has finitely many subgroups, then \( G \) is a finite group.

2. (No proofs necessary for this problem, but show your work.)
(a) Draw the Cayley digraph for \( \mathbb{Z}_8 \) that comes from the generating set \( S = \{2, 5\} \).
(b) Compute the order of the following permutation (which is written in 2-line notation):
\[
\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 3 & 1 \end{pmatrix} \in S_5.
\]
(c) Write the following product of cycles (in \( S_6 \)) in 2-line notation: \((26)(12)(53)(34)(1264)\).
(d) List all homomorphisms \( \mathbb{Z}_6 \to S_3 \).
(e) List all homomorphisms \( \mathbb{Z}_4 \to D_4 \).

3. Prove that \( S_n \) is non-abelian for all \( n \geq 3 \).

4. Section 7 # 10

5. Section 8 # 4, 8, 35, 44, 49